10. How many ways are there to choose, with repetition allowed, r objects from n distinguishable objects?

The solution to #10 is the same as the solution to #1, which is a very easy Mississippi problem. We notice that the the solution $\frac{(n-1+r)!}{r!(n-1)!}$ can be written $\binom{n+r-1}{r}$ and thus we have the following.

Observation. With repetition allowed, the number of ways to choose r objects from n distinguishable objects is

$$\binom{n+r-1}{r}.$$

The formula in the observation above is well worth remembering, assuming that you know the meaning of $\binom{n+r-1}{r}$. With this observation, we have the contents of Table 1.2.

	Ordered Selections (Permutations)	Unordered Selections (Combinations)
Without Repetition	$\frac{n!}{(n-r)!}$	$\binom{n}{r}$
With Repetition (allowed)	n^r	$\binom{n+r-1}{r}$

TABLE 1.2. Selecting r from n Distinguishable Things.

Some comments about notation are in order. We read "$\binom{n+r-1}{r}$" as "n choose r with repetition." Let's be quite clear that in "selecting with repetition" we mean that repetition is *allowed* in making the selection and not that a repetition is *required* in the choice. If we select with repetition 3 pieces of fruit from oranges, bananas, peaches, and apples, we could select 3 oranges or we could select 1 orange, 1 banana, and 1 peach. How many possible possible choices are there in this case? Although each of the answers 20, $\binom{6}{3}$, and $\binom{4+3-1}{3}$ is correct, we prefer, for pedagogical reasons, the last form since this form carries the most information. We also note that, although the language is different, "picking with replacement" is mathematically the same thing as "picking with repetition." Surely, the number of ways to pick 5 cards from a deck, if after each pick the card is replaced in the deck, is $\binom{52+5-1}{5}$. The language for picking ice cream cones seems to require "with repetition" and not "with replacement."