MAT 3701: HW Assignment Template   My Name

Hi! This is one of my favorite resources for math symbols:
https://en.wikibooks.org/wiki/LaTeX/Mathematics.

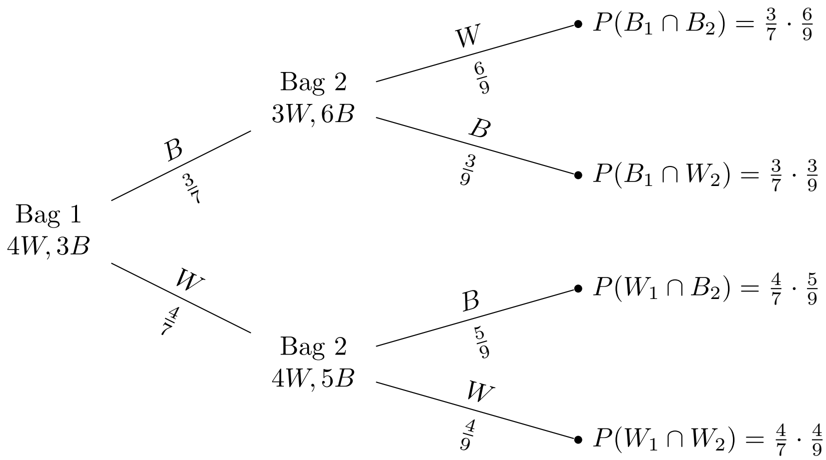
1.15 In how many ways can six tosses of a coin yield two heads and four tails?

SOLUTION: By Theorem 1.7,

$$\binom{6}{2} = \frac{6!}{2! \cdot 4!} = 15.$$

1.1 Here is an example of a probability tree, more examples can be found here,
http://www.texample.net/tikz/examples/feature/trees/

SOLUTION:



2.1 (a)

(b)

1. Prove by induction. Show that for any $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i = \frac{n\,(n+1)}{2}.$$

Proof. Base case $n = 1$: If $n = 1$, the left side is 1 and the right side is $\frac{1(2)}{2} = 1$. So, the statement holds when $n = 1$.

Inductive hypothesis: Suppose the statement holds for some n. We want to show true for $n + 1$, i.e.,

$$\sum_{i=1}^{n+1} i = \frac{(n+1)\,(n+2)}{2}.$$

Inductive step:

$$\begin{aligned}\sum_{i=1}^{n+1} i &= (n+1) + \sum_{i=1}^{n} i \\ &= (n+1) + \frac{n(n+1)}{2}, \text{ by our inductive hypothesis} \\ &= \frac{2(n+1)}{2} + \frac{n(n+1)}{2} \\ &= \frac{2(n+1) + n(n+1)}{2} \\ &= \frac{(n+1)(n+2)}{2}\end{aligned}$$

which is our right side. So, the statement holds for $n+1$. By the principle of mathematical induction, the statement holds for all $n \in \mathbb{N}$. □

2. Find numbers a, b, and c, so that $\displaystyle\int_3^7 e^{(2x+1)^2}\,dx = c\int_a^b e^{u^2}\,du$.

   SOLUTION: Let $u = 2x+1$. Then $du = 2\,dx$, ie, $dx = \frac{du}{2}$. Hence,

$$\int_{x=3}^{x=7} e^{(2x+1)^2}\,dx = \frac{1}{2}\int_7^{15} e^{u^2}\,du.$$

6.3 We will encounter this integral in chapter 6, $\int_0^\infty x^n e^{-x}\,dx = n!$.

Can you solve

$$\int_0^1 \int_0^1 x^2 y^2\,dx\,dy?$$