MAT 2420: Probability Problem Set II **Due Monday, October 8**

Explain all answers completely in well-organized full sentences! Type or write neatly.

1. A fair coin is tossed until the first head is obtained.

   (a) What is the probability that only one toss is required?

   (b) What is the probability that two tosses are required?

   (c) What is the probability that three tosses are required?

   (d) What is the probability that ten tosses are required?

   (e) What is the probability that 31 tosses are required?

   (f) Let X be the number of tosses required. What is the probability that $X = 11$?

2. A fair cubical die is tossed until a "1" is obtained.

   (a) What is the probability that only one toss is required?

   (b) What is the probability that two tosses are required?

   (c) What is the probability that three tosses are required?

   (d) What is the probability that ten tosses are required?

   (e) What is the probability that 31 tosses are required?

   (f) Let X be the number of tosses required. What is the probability that $X = 11$?

3. Assume a light bulb is left on all the time and is equally likely to burn out at any moment. (That is, the bulb does not "age" or deteriorate; the chance that it burns out during a given interval, if it was shining at the beginning of the interval, is proportional to the length of the interval. This model is fairly realistic, at least for incandescent bulbs.) Suppose the bulb has a 36% chance of burning out in the first month after it is turned on.

   (a) What is the probability that the bulb will burn out in two months? (Hint: it can either burn out in the first month, or survive the first month but then burn out in the second.)

   (b) In half a month? (Be careful! The answer is *not* 18%.)

4. Suppose there is a test for a certain type of cancer with the property that 90% of those with cancer react positively, whereas 5% of those without cancer react positively (false positives). In a certain region, 1% of the population has this type of cancer. If people from this region are tested at random, what percentage will get a positive test result?

5. For the cancer test and population above, suppose a person chosen at random gets a positive test result. What is the probability that person actually has cancer? What is the probability that the result is a false positive, and the person does not actually have cancer?

6. In a certain town there are three auto mechanics, Albert, Betty, and Cathy. Albert makes the correct repair on 50% of the jobs given to him. Betty makes the correct repair on 80% of the jobs given to her. Cathy makes the correct repair on 30% of the jobs given to her.

   Since Betty has a good reputation, she gets 60% of the repair jobs. Albert gets 30% because, although he is not so good a mechanic, he is cheap. Cathy gets 10% of the jobs.

   A job is chosen at random from the records of auto repair jobs done in the town over the past year.

   (a) What is the probability that the repair was done correctly?

   (b) If the repair was correct, what is the probability that Albert did it?

   (c) If the repair was correct, what is the probability that Betty did it?

   (d) If the repair was correct, what is the probability that Cathy did it?

   (e) If the repair was incorrect, what is the probability that Albert did it?

   (f) If the repair was incorrect, what is the probability that Betty did it?

   (g) If the repair was incorrect, what is the probability that Cathy did it?

7. The half-life of a certain radioactive element is 8 days. This means that at the end of any 8 day period, half of the atoms of the substance that were present at the beginning of that period will have decayed to another element.

   (a) What is the probability that an atom of the substance, selected initially at random, has decayed after 16 days?

   (b) After 4 days?

(c) What is the probability it decays between day 4 and day 8?

(d) Between day 8 and day 16?

8. Suppose there are three chests each having two drawers. The first chest has a gold coin in each drawer, the second chest has a gold coin in one drawer and a silver coin in the other drawer, and the third chest has a silver coin in each drawer. A chest is chosen at random and a drawer opened. If the drawer contains a gold coin, what is the probability that the other drawer also contains a gold coin? (Be very, very careful!)

9. A classic game show works has follows: There are three curtains. Behind one curtain is a fabulous prize. Behind the other two curtains are goats. The contestant picks a curtain at random. To create excitement, the host opens one of the other curtains, revealing a goat, and asks, "Do you want to stick with your first choice, or switch to the remaining curtain"? What is the best strategy for the contestant?

10. What is the probability that, out of the 32 students in this class, two have the same birthday? (Assume for simplicity that there are 365 days in a year, with no leap years, and that a person is equally likely to be born on any day of the year.)

11. Two fair six-sided dice are rolled.

    (a) What is the probability that the sum of the two dice is 2?

    (b) What is the probability that the sum of the two dice is 3?

    (c) What is the probability that the sum of the two dice is 4?

    Let X be the value of the first die. Let Y be the value of the second. Let $Z = X+Y$.

    (d) What is the probability that $X = 2$?

    (e) What is the probability that $Y = 3$?

    (f) What is the probability that $Z = 5$?

    (g) Compute the probabilities for $Z = 6$, $Z = 7$, $Z = 8$, $\ldots Z = 12$.