Eastern Illinois Integrated Conference in Geometry, Dynamics, and Topology: Abstracts

April 15-17, 2016

Chris Leininger (UIUC)– Strict contractions for 3-dimensional hyperbolic manifolds.

Friday, April 15 at 4:00pm in Room 1205.

The Schwarz-Pick Theorem allows one to construct strict contractions between hyperbolic surfaces which have positive degree. I will discuss work with Grant Lakeland, building on a construction suggested by Ian Agol, providing the first examples of this phenomenon in dimension 3. Through the work of Gueritaud-Kassel and Tholozan, this has some interesting connections to locally homogeneous spaces and volumes, which I will also briefly describe.

Priyam Patel (Purdue)– Lifting curves simply in finite covers of hyperbolic surfaces.

Saturday, April 16 at 9:00am in Room 1205.

It is a well known result of Peter Scott that the fundamental groups of surfaces are subgroup separable. This algebraic property of surface groups also has important topological implications. One such implication is that every closed curve on a surface lifts to a simple closed curve in a finite cover of the surface (i.e. lifts simply). A natural question that arises is: what is the minimal degree of a cover in which a given closed curve lifts simply? We will begin this talk by discussing various results answering the above question for hyperbolic surfaces. In particular, we will focus on recent joint work with T. Aougab, J. Gaster, and J. Sapir answering this questions for cusped hyperbolic surfaces.

Brian Bourne (EIU)– The use of persistent homology to quantify molecular shape similarity.

Saturday, April 16 at 10:00am in Room 1205.

I will give an overview on the use of methods from pesistent homology to questions about shape similarity, particularly in regards to molecular shape. Results of the initial treatments will be reviewed, and we will also discuss ongoing work.

Kwok Hao Lee (Wash. U.)– Law and order: group actions on $\mathbb{R}$.

Saturday, April 16 at 10:00am in Room 2120.

Left orderings on groups are closely related to some interesting open questions in low-dimensional topology, since they are closely related to group actions on the real line. In this talk, I survey some key results in the theory of ordered groups and in that of said group actions.

Christopher Bouska (Kansas State)– Cheeger constants of hyperbolic surfaces.

Saturday, April 16 at 10:30am in Room 1205.

For a given surface, how does the geometry of the surface affect the audible frequencies given by the surface when it is struck by a hammer? The Cheeger constant of the surface provides an answer to this question and is defined as follows. First, one considers splitting the surface into two (possibly disconnected) subsurfaces with a multi-curve boundary and then measures the ratio of the length of this boundary to the area of the lesser-area subsurface. The Cheeger constant is taken to be the infimum of these ratios over all possible splittings. Although the Cheeger constant is geometric, it has a reputation for being difficult to compute directly. Since there are many examples of surfaces for which the Cheeger constant can be studied, it is natural to first restrict to those having finite area and constant sectional curvature. Given these restrictions, a consequence of the Gauss-Bonnet Theorem is the surface must be hyperbolic (negatively curved) when the genus is greater than one. As a result, we will focus the talk on these surfaces and discuss recent progress that has been made in directly computing their Cheeger constants. This is joint work with Brian Benson and Grant Lakeland.

Milana Golich (Purdue)– Cyclic evasion in the four bug problem.

Saturday, April 16 at 10:30am in Room 2120.

We consider the four bug problem where bug j evades bug $j+1$. Based on our experimental and theoretical findings, we have determined that all four bug configurations converge to a stable configuration of either a square or a self-intersecting line. However, a slight perturbation of the square can cause the system to enter a limiting cycle where the configuration does not converge to a stable configuration in finite time.

Grace Work (UIUC)– Transversals to horocycle flow on the moduli space of translation surfaces.

Saturday, April 16 at 11:20am in Room 1205.

Computing the distribution of the gaps between slopes of saddle connections is a question that was studied first by Athreya and Cheung in the case of the torus, motivated by the connection with Farey fractions, and then in the case of the golden L by Athreya, Chaika, and Lelievre. Their strategy involved translating the question of gaps between slopes of saddle connections into return times under horocycle flow on the space of translation surfaces to a specific transversal. We show how to use this strategy to explicitly compute the distribution in the case of the octagon, the first case where the Veech group had multiple cusps, how to generalize the construction of the transversal to the general Veech case (both joint work with Caglar Uyanik), and how to parametrize the transversal in the case of a generic surface in $\mathcal{H}(2)$.

Witsarut Pho-on (UIUC)– Infinite unicorn paths and Gromov boundaries.

Saturday, April 16 at 11:20am in Room 2120.

I will provide direct elementary proofs of results of Klarreich and Schleimer identifying the Gromov boundaries of the arc and curve graph and the arc graph, respectively. The proofs use the tool called unicorn paths, developed by Hensel, Przytycki and Webb in their elementary proofs of hyperbolicity of the arc and curve graph and the arc graph. More precisely, I extend the notion of unicorn paths between two arcs to the case where one arc is replaced by a bi-infinite geodesic asymptotic to a lamination. Using these modified unicorn paths, I define homeomorphisms from some spaces of laminations to the Gromov boundaries of the arc and curve graph and the arc graph which are also equivariant under mapping class groups.

Rachel Roberts (Wash. U.)– Cousins: foliations and contact structures.

Saturday, April 16 at 1:40pm in Room 1205.

I will give an overview of contact structures and codimension one foliations in 3-manifolds, emphasizing how these objects are related.

Jeffrey Meier (Indiana)– A new approach to knot theory in dimension four.

Saturday, April 16 at 3:00pm in Room 1205.

This talk will be an introduction to the theory of bridge trisections, which represents a new approach to the study of knotted surfaces in the four-sphere and gives a four-dimensional analogue to the concept of a bridge splitting of a knot or link in the three-sphere. A bridge trisection allows a knotted surface to be described by a triple of trivial tangles whose pairwise unions are all unlinks. From this, a diagrammatic representation of the surface can be given, and a complete calculus of moves required to relate such diagrams can be described. A main hope is that the theory will allow for three-dimensional techniques and invariants from knot theory to be promoted to the study of knotted surfaces, and connections to knot concordance and the braid group will be described. The study of knotted surfaces is riddled with important unsolved problems: Are unknotted surfaces characterized by the fundamental group of their complements? Do knotted surfaces admit unique prime decompositions? Can surgery operations on knotted surfaces in the four-sphere produce exotic four-manifolds? The theory of bridge trisections may offer new approaches to these intriguing questions.

Anton Lukyanenko (Michigan)– Quasi-crystals in non-commutative spaces.

Sunday, April 17 at 9:00am in Room 1205.

Quasi-crystals are certain types of discretizations of Euclidean space that arise naturally in applications and have properties analogous to those of crystals (or lattices). We study discretizations of nilpotent Lie groups, showing first that these can be quite unpleasant: they need not be perturbations of lattices in either the bounded-displacement or bi-Lipschitz sense. We then define quasi-crystals in these groups and show that they are much better-behaved and are, as in the Euclidean case, (almost always) perturbations of lattices. This is joint work with Tullia Dymarz, Sean Li, and Michael Kelly.

Mark Pengitore (Purdue)– Effective separability of finitely generated nilpotent groups.

Sunday, April 17 at 10:00am in Room 1205.

In this talk, we give the precise asymptotic behavior of residual finiteness for finitely generated nilpotent groups. Similarly, we give polynomial upper and lower asymptotic bounds for conjugacy separability for finitely generated nilpotent groups.

Nicholas Miller (Purdue)– Arithmetic progressions in the primitive length spectrum.

Sunday, April 17 at 11:10am in Room 1205.

There have been a host of prime geodesic theorems over the past several decades displaying a surprising analogy between the behavior of primitive, closed geodesics on hyperbolic manifolds and the behavior of the prime numbers in the integers. For instance, just as the prime number theorem dictates the asymptotic growth of the number of primes less than n, there is an analogous asymptotic growth for primitive, closed geodesics of length less than n. In this talk, I will give a brief review of the relevant definitions and go on to give the history of this analogy. I will then discuss some recent work extending this relationship to give the geodesic analogue of the GreenTao theorem on arithmetic progressions in the prime numbers.

Peter Shalen (UIC)– Volume and homology for hyperbolic 3-orbifolds, and the enumeration of arithmetic groups.

Sunday, April 17 at Noon in Room 1205.

A theorem of Borel's asserts that for any positive real number V, there are at most finitely many arithmetic lattices in $\mathrm{PSL}_2(\mathbb{C})$ of covolume at most V, or equivalently at most finitely many arithmetic hyperbolid 3-orbifolds of volume at most V. Determining all of these for a given V is algorithmically possible for a given V thanks to work by Chinburg and Friedman, but appears to be impractical except for very small values of V, say $V = 0.41$. (The smallest covolume of a hyperbolic 3-orbifold is about 0.39.) It turns out that the difficulty in the computation for a larger value of V can be dealt with if one can find a good bound on $\dim H_1(O, \mathbb{Z}/2\mathbb{Z})$, where O is a hyperbolic 3-orbifold of volume at most V.

In the case of a hyperbolic 3-manifold M, not necessarily arithmetic, joint work of mine with Marc Culler and others gives good bounds on the dimension of $H_1(M, \mathbb{Z}/2\mathbb{Z})$ in the presence of a suitable bound on the volume of M. In this talk I will discuss some analogous results for hyperbolic 3-orbifolds, and the prospects for applying results of this kind to the enumeration of arithmetic lattices. A feature of the work that I find intriguing is that while it builds on my geometric work with Culler, the new ingredients involve primarily purely topological arguments about manifolds—the underlying spaces of the orbifolds in question—and have a classical, combinatorial flavor.

At this point, although the work is only partially written up, it appears that I can prove the following statement:

If Ω is a hyperbolic 3-orbifold of volume at most 1.72, having a link as singular set and containing no embedded turnovers, then

$$\dim H_1(\Omega;_2) \leq 1 + \max\left(3, 7\left\lfloor \frac{10}{3}(\Omega)\right\rfloor\right) + \max\left(3, 7\left\lfloor \frac{5}{3}(\Omega)\right\rfloor\right).$$

In particular, $\dim H_1(\Omega;_2) \leq 50$. Various stronger bounds on $\dim H_1(\Omega;_2)$ follow from stronger bounds on the volume of Ω.

The restriction on turnovers is not an obstruction to applying the results to the enumeration of arithmetic groups. The assumption that the singular set is a link is more serious, but as it is used only in a mild way in this work, the methods seem promising for the prospective application.