ECN 3971
Homework Answers for 2nd exam

5.14

a.)   We drew a cdf in class.

b.) $P(X \geq 5) = 0.08 + 0.10 + 0.12 + 0.08 + 0.10 = 0.48$

c.) $P(3 \leq X \leq 7) = 0.15 + 0.12 + 0.08 + 0.10 + 0.12 = 0.57$

5.19

c.) $P(X \geq 3) = 0.28 + 0.15 + 0.07 = 0.50$

d.) $E(X) = 0(0.10) + 1(0.14) + 2(0.26) + 3(0.28) + 4(0.15) + 5(0.07) = 0 + 0.14 + 0.52 + 0.84 + 0.60 +$
        $0.35 = 2.45$

e.) $\sigma_x^2 = (0 - 2.45)^2(0.10) + (1 - 2.45)^2(0.14) + (2 - 2.45)^2(0.26) + (3 - 2.45)^2(0.28) +$
        $(4 - 2.45)^2(0.15) + (5 - 2.45)^2(0.07) = 0.60025 + 0.29435 + 0.05265 + 0.0847 + 0.360375$
        $+ 0.455175 = 1.8475$

        $\sigma_x = 1.359$

5.40

$\pi = 0.4$
$1 - \pi = 0.6$
$n = 5$

a.) $P(X = 5) = C_5^5 (0.4)^5(0.6)^0 = (1)(0.01024)(1) = 0.01024$

b.) $P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

        $P(X = 3) = C_3^5(0.4)^3(0.6)^2 = [5!/3!2!](0.064)(0.36) = 10(0.064)(0.36) = 0.2304$

        $P(X = 4) = C_4^5(0.4)^4(0.6)^1 = [5!/4!1!](0.0256)(0.6) = 5(0.0256)(0.6) = 0.0768$

        $P(X \geq 3) = 0.2304 + 0.0768 + 0.01024 = 0.31744$

c.) $n = 4$

        $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

        $P(X = 2) = C_2^4(0.4)^2(0.6)^2 = [4!/2!2!](0.16)(0.36) = 6(0.16)(0.36) = 0.3456$

        $P(X = 3) = C_3^4(0.4)^3(0.6)^1 = [4!/3!1!](0.064)(0.6) = 4(0.064)(0.6) = 0.1536$

        $P(X = 4) = C_4^4(0.4)^4(0.6)^0 = [4!/4!0!](0.0256)(1) = (1)(0.0256)(1) = 0.0256$

        $P(X \geq 2) = 0.3456 + 0.1536 + 0.0256 = 0.5248$

d.) $E(X) = n\pi = 5(0.4) = 2$

e.) $E(X) = n\pi + 1 = 4(0.4) + 1 = 1.6 + 1 = 2.6$

5.56

N = 16
n = 8
S = 8

$P(X = 4) = \{C_4^8 \times C_4^8\} / C_8^{16} = \{[8!/4!4!] \times [8!/4!4!]\} / [16!/8!8!] = \{(70) \times (70)\}/12870 = 4900/12870 = 0.3807$

<u>5.58</u>

N = 10
n = 6
S = 5

$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$P(X = 0) = 0$

$P(X = 1) = \{C_1^5 \times C_5^5\} / C_6^{10} = \{[5!/4!1!] \times [5!/5!0!]\} / [10!/6!4!] = \{5 \times 1\} / 210 = 0.0238$

$P(X = 2) = \{C_2^5 \times C_4^5\} / C_6^{10} = \{[5!/3!2!] \times [5!/4!1!]\} / [10!/6!4!] = \{10 \times 5\} / 210 = 0.238$

$P(X < 3) = 0.2619$

<u>5.64</u>

$\lambda = 3$

$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$P(X = 0) = \{(e^{-3})(3)^0\}/0! = 0.049787$

$P(X = 1) = \{(e^{-3})(3)^1\}/1! = (0.049787)(3) = 0.1493612$

$P(X = 2) = \{(e^{-3})(3)^2\}/2! = \{(0.049787)(9)\}/2 = 0.2240418$

$P(X \leq 2) = 0.42319$

<u>5.70</u>

n = 6,000
$\pi = 0.001$

$n\pi = 6$

$P(X \geq 3) = 1 - P(X < 3)$

P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)

P(X = 0) = {(e^{-6})(6)^0}/0! = 0.00248

P(X = 1) = {(e^{-6}(6^1)}/1! = (0.00248)(6) = 0.01487

P(X = 2) = {(e^{-6}(6^2)}/2! = {0.00248)(36)}/2 = 0.04462

P(X < 3) = 0.06197

P(X ≥ 3) = 1 – 0.06197 = 0.93803

6.17

a.)   0.8849

b.)   0.0918

c.)   0.0446

d.)   0.8413

e.)   0.0233

f.)   0.8403

g.)   0.1141

6.22

a.)   0.6554

b.)   0.6554

c.)   In the first case, Z = 0.4. In the second case, Z = -0.4. Because of symmetry, P(Z < 0.4) = P(Z > 0.4).

d.)   P(-1.6 < Z < 0.4) = 0.6554 – 0.0548 = 0.6006

e.)   1.28 = (X– 380)/50   X = 444

        -1.28 = (X– 380)/50   X = 316