JOB RESPONSIBILITY, PAY AND PROMOTION*

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How are pay and promotion prospects related to job responsibility? A job entails responsibility to the extent that the value of the job outcome is sensitive to the worker’s input of effort. In my model, an employer uses termination contracts to elicit effort from workers. The optimal wage increases with responsibility. I show that the employer can reduce incentive costs by structuring a job ladder and offering workers a self-enforcing prospect of promotion. In fact, the employers will choose to pay differentiated wages to identical workers in identical jobs, promoting workers from the lower-paying to the higher-paying positions as vacancies occur.

I. RESPONSIBLE JOBS

A responsible job is one in which the value of job outcomes is highly sensitive to the input of worker effort. The responsible worker is not closely monitored during the production process, but the outcome of his work is evaluated after the fact, and credit or blame is assigned at that time. The degree of responsibility may be measured by the variation in the value of job outcomes over the feasible range of worker effort. This variation is quite different from a worker’s marginal product. As with ship captains and civil engineers, the magnitude of losses caused by a single failure of a responsible worker may, in the extreme, be many times his expected lifetime marginal product or income.

Some responsible jobs are held by highly skilled professionals; others are held by relatively unskilled workers. Business executives have responsible jobs: their strenuous efforts can bring substantial profits, their lapses can drive a prosperous firm into bankruptcy. Brain surgeons, bus drivers and firemen all have responsible jobs, because their failures can lead to very high costs, including loss of life. Many factory workers, such as equipment maintenance technicians, machine operators, dispatch personnel, among others, also have responsible jobs.

The level of responsibility is not necessarily reflected in the tasks that a worker must perform. Consider, for example, two different jobs, each associated with the following task: the worker in each job must watch a control panel. If a red light comes on, the worker must throw a switch. That’s all. What differs in the two jobs is not the tasks but the consequences of failure. In the first case, let us suppose, the job is in a nuclear facility: failure will result in a nuclear meltdown. In the second case, the job is in a fast-food restaurant: failure will result in a dozen burnt hamburgers. An important implication of this paper is that despite the workers’ identical qualifications and identical assigned tasks, it makes sense to pay the first worker a lot more than the second is paid. We shall try to understand how much more and why.

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The productive environment of responsible employees differs both from the environment of ordinary employees and from that of independent contractors. A worker in a non-responsible position normally works under direct supervision, and his effort level is observed by the employer, albeit somewhat imperfectly. If his inputs are appropriate he is rarely held accountable for the consequences of his actions. In contrast to this, an independent contractor is committed to produce a product or service described in an explicit contract, and little attention need be paid to his input of effort. Satisfactory completion of the contractor’s stipulated output can, in principle, be verified by third parties and is subject to legal enforcement mechanisms.

The responsible job represents the worst of both worlds with respect to information availability and contract enforceability. It is in the nature of a responsible position that direct supervision of the worker is impractical, so that the employer does not observe his effort level. Furthermore, although the employer can evaluate the worker’s performance after the fact, the quality of that performance cannot be verified by third parties or form the basis of an enforceable contract. In part, this is because the standards applied by the employer are difficult to state explicitly, especially in advance. As Fairburn and Malcomson (1994) put it: ‘It is not that output is unobserved, but that output is too complex or multidimensional to write into a contract in an unambiguous way. Firm and worker know what performance level is achieved, but this is too difficult or costly for a court to evaluate.’

To sum up, the concept of job responsibility is defined in this paper as follows:

**Definition 1.** A responsible job is one for which: (i) worker effort is not monitored directly, (ii) the value of job outcomes is highly sensitive to the input of worker effort, and (iii) an employment contract contingent on job performance would not be legally enforceable.

**Definition 2.** The degree of job responsibility is an index that measures the variation in the value of job outcomes over the feasible range of worker effort.

The problem of providing incentives to workers with unobservable inputs and unverifiable job outcomes has been examined at length in the literature. The most obvious incentive mechanisms can be ruled out for responsible jobs. For example, employers will not be able to motivate responsible workers with simple promises of future payments in compensation for good current performance, because such promises would be subject to moral hazard. The posting of bonds by workers, or equivalently, promises of rising wage profiles after an initial period of below-market wages, can be eliminated for the same reason. Nor will employers be able to induce effort by extracting commitments from workers to pay for losses after they occur. But economists have proposed a number of employment contracts that provide appropriate incentives and avoid the moral hazard problem created by unverifiable outcomes. Most of these contracts fall into two general categories: self-enforcing termination contracts and labour tournaments.

A termination contract provides a worker with an above-market efficiency
wage rate that in itself is outcome-independent, but it terminates employment if the outcome fails to meet a designated standard. The worker will wish to deliver a good performance so as to avoid the discontinuance of his above-market wage. Inasmuch as firing a worker, who must then be replaced, is not intrinsically advantageous to the firm, the employer’s moral-hazard problem is removed and the employment contract becomes self-enforcing. Termination contracts are analysed in Malcomson (1981), Stoft (1982), and Shapiro and Stiglitz (1984). Bull (1987) examines a somewhat different case in which reputation within the firm, rather than a termination mechanism, is used to overcome moral hazard. In the context of a repeated game, MacLeod and Malcomson (1989) completely characterise a broad class of self-enforcing labour contracts, of which termination contracts and the Bull mechanism are special cases.

A labour-tournament contract surmounts the problem of non-verifiable outcomes by an entirely different route. This type of contract contains a legally enforceable specification of an aggregate wage-bill that depends only on verifiable data. The firm rewards individual performances after the fact by dividing the specified total among the workers, with the largest rewards going to the best performers. Because aggregate costs are fixed, the firm’s moral hazard problem is removed. Labour-tournament contracts are described and analysed by Lazear and Rosen (1981) and Malcomson (1986), among others. This type of contract may cause excess competition between workers and hinder worker cooperation. It is also vulnerable to worker bribery or other ‘influence activities’ of the sort described by Milgrom (1988). For example, in return for being designated as a high performer by the firm, a poorly performing worker may offer to refund privately some part of his reward. Such possibilities may make the firm’s payment structure suspect. Fairburn and Malcomson (1994) discuss these problems and show how they may be ameliorated.

The purpose of this paper is to elucidate the relationship between the degree of responsibility, pay, and promotion opportunities, on a theoretical level. It would have been highly desirable to develop general results valid for all feasible labour contracts, but I have not attempted to do so here (nor have I seen studies of so general a scope in the literature). Rather, I have chosen to restrict the context of this analysis to that of the self-enforcing termination contract. I made this choice for several reasons. The idea of the termination contract is straightforward and well understood. Unlike the tournament, the termination contract involves only the employer and the worker in question and does not require a legal enforcement mechanism. The termination contract is an appropriate framework for an exploration of job responsibility, inasmuch as termination or demotion is a likely consequence of failure for many responsible employees. And, perhaps most important, I am able to obtain quantitative results about wages and economic rents within this restricted domain.

In Section II, I analyse the properties of self-enforcing termination contracts

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1 A general review and synthesis of the efficiency-wage literature is provided by Weiss (1990). Also, see the introduction of Akerlof and Yellen (1986), Stiglitz (1987), and Solow’s AEA Presidential Address (1986).
and examine the relation between their parameters and the specified degree of responsibility. Using the framework of a dynamic game between an employer and a sequence of workers, I develop a model of such contracts. In this model, the employer hires one worker at a time into a job that is subject to failures. The employers offers each worker a contract chosen from a parametrised family of contracts. The worker responds by choosing his level of effort and the failure rate, which are inversely related. The employer may or may not terminate the worker when a failure occurs, but if she does dismiss him, she will have to replace him immediately, possibly at a cost to herself.

In general, this game will have a ‘tough’ equilibrium in which the employer always punishes failure with termination. In tough equilibria, workers in responsible jobs receive rents as compared with identical workers in non-responsible jobs. These are true rents, above and beyond compensation for additional effort expended. They form an addition to the Ricardian rents that infra-marginal workers may receive. It turns out that the aggregate value of worker rents will be closely related to the aggregate value of realised losses arising from failures in jobs of the same type (e.g. Hollywood movie directors would receive high rents). This is because a high level of realised losses implies a combination of high costs of failure and substantial difficulties in preventing those failures from occurring. We show that it is the simultaneous presence of these two factors that yields rents to responsible employees.

If the game is structured so that the employer is able to commit to a worker-dismissal policy, there is a unique subgame-perfect equilibrium. If worker-replacement costs are sufficiently small as compared with the cost of failure, then the unique equilibrium is tough. I will show that the employer’s level of utility in the tough equilibrium equals or exceeds her level of utility in any other equilibrium. This implies that a commitment to a tough dismissal policy can only help the employer. In the context of a responsible job with non-verifiable outcomes and worker replacement costs, however, commitment to a tough dismissal policy surely would be hard to achieve. But I show that even when commitment is impossible, there are trigger-strategy equilibria that have the same play and give the employer the same level of utility as does the tough equilibrium. This suggests that even in the absence of the ability to commit, the employer may be able to achieve credibility for a tough policy.

In Section III, I explore the relationship of promotion prospects to job responsibility and associated remuneration. I show that employers can reduce their costs by structuring job ladders and promoting from within. This is because the combination of termination contracts and job ladders creates what might be called economies of scope in providing incentives: a high wage in a job high on the promotion ladder not only elicits more effort from the incumbent in that position, but also stimulates workers in jobs lower down, provided only that they have a positive probability of promotion to the high-paying position. In fact, it is in the interest of an employer to pay differentiated wages to identical workers with identical degrees of responsibility in identical jobs, promoting workers from the lower-paying to the higher-paying positions as vacancies occur.
Consider an economy in which all workers of a given type may be employed in either responsible or non-responsible jobs. The workers are identical, risk-neutral and have infinite lifetimes. Let \( z \) denote a worker’s rate of remuneration, and \( s \) his level of on-the-job effort. For simplicity, we normalise to 0 the required level of effort and market-clearing wage rate of non-responsible jobs. We represent the worker’s current utility by the expression \( z - s \), which implies that non-responsible jobs provide zero utility. Thus, the value of utility for a worker in a responsible job may be viewed as a measure of the net rent provided by that job as compared with non-responsible job alternatives. The number of responsible jobs is assumed to be relatively small, so that a worker who loses a responsible job has only a negligible probability of obtaining another one.

A particular employer has one responsible job that must be filled by one worker at all times. This job will have identical characteristics for the infinite future. The employer is indifferent between the many workers available to fill a job vacancy.

The worker’s job performance is subject to randomly occurring failures, generated by a Poisson process. The mean failure rate or hazard rate \( \phi \) is controlled by the worker through his input of effort \( s \). The level of effort required to maintain a particular failure rate is given by the function \( s = s(\phi) \), which is technologically determined and exogenous. This required-effort function is assumed to be positive, downward-sloping and asymptotic to both axes, so that a near total lack of effort results in almost immediate failure, while no finite amount of effort can render a failure impossible. Additional regularity conditions are imposed on \( s \) in the Appendix.

The employer is unable to observe the level of worker effort directly but does observe worker failures when they occur. Each failure causes the employer to sustain a loss of a specified magnitude, \( L \). If a failure occurs, the employer may choose to fire the incumbent worker and replace him with a new but identical worker. Firing and replacing an incumbent worker cost the employer an amount given by \( F \geq 0 \).

It is natural to model the process described above as a dynamic game between the employer and a sequence of workers. The dynamic game is composed of a sequence of identical stage games. Each stage game represents the interaction between the employer and an individual worker. In order to focus on the informational limitations associated with job responsibility (and to facilitate solution of the model), I model the stage game as a one-shot game in extensive form. This structure excludes non-stationary employer strategies that yield upward sloping wage profiles (see discussion in Section III). It also disallows employer strategies in which the probability of worker dismissal after a failure depends on the number, frequency, or timing of the worker’s previous failures.

In order to make subgame perfection a useful solution-concept, we make the innocuous assumption that although the employer cannot observe the effort level of an incumbent worker, she knows the effort level of previous workers.

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failures. However, I argue below that in a more general model, such strategies, would not be supported in equilibrium.

Efficiency-wage models differ from standard principal-agent models in an important way. In most principal-agent models, the agent earns his reservation utility in equilibrium, so that if the principal wants to increase the punishment associated with bad outcomes, she must compensate the agent with higher payments for good outcomes. But in an efficiency-wage framework like this one, the worker would earn positive economic rents in equilibrium, so that punishment for bad outcomes could be increased without a *quid pro quo*. In particular, if the probability of dismissal after the first failure were less than one, the employer would be able to increase it while holding the wage rate fixed without pushing the worker below his reservation utility. Because the worker’s choice of an effort level is an increasing function of the probability of dismissal, the increased punishment would have a positive incentive effect. Therefore, for sufficiently small worker replacement costs, no strategy with a probability of dismissal less than one can be an employer’s best response, even when that employer is in a position to use information about the worker’s past behaviour.

I now proceed to develop the game in two variants. In the first, I suppose that the employer can commit to a worker-dismissal policy at the time each worker is hired. I use this variant to develop most of the machinery needed to solve the second variant, in which the employer cannot precommit to a worker-dismissal policy. Furthermore, I show that availability of employer commitment yields a unique subgame-perfect equilibrium with an employer utility level that forms an upper bound on equilibrium utility levels when no commitment is possible. Then, in Section II.B, I show that even without having the ability to commit, the employer can attain the same equilibrium behaviour, and thus the same utility level, that commitment would imply.

II. A. The Responsibility Game with Employer Commitment

In this game variant, the stage game between the employer and the individual workers has three nodes:

(i) The employer chooses a wage rate, \( z > 0 \), for the current worker, to be paid continuously during his tenure, and a probability of dismissal, \( \alpha > 0 \), to be applied after each failure.\(^3\)

(ii) Given \( z \) and \( \alpha \), the worker selects a failure rate, \( \phi \), (and thus a level of effort, \( s(\phi) \)) to be applied continuously during his tenure.

(iii) Given the failure rate \( \phi \) and the firing policy \( \alpha \), nature randomly selects the time \( \tau \) when the worker is to be fired. At that time the employer must pay the replacement cost \( F \).

The employer’s moves in each stage game are pairs of the form \( (z, \alpha) \) where \( z > 0 \) and \( 0 < \alpha \leq 1 \), so that her action space may be represented by the set \( A = (0, \infty) \times (0, 1] \). The employer’s strategy space consists of all functions mapping histories of play to a current action \( (z, \alpha) \in A \). We will say a strategy

\(^3\) We rule out \( z = 0 \) and \( \alpha = 0 \) without loss of generality; it will become apparent that neither \( z = 0 \) nor \( \alpha = 0 \) would be an equilibrium choice.

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is tough on a given set of nodes if \( \alpha = 1 \) at each member of the set. Otherwise, we will say that the strategy is lenient.

The worker’s strategy space consists of all functions \( \Phi \) that map the history of previous stage games and the employer’s current action, \((z, \alpha)\), to a failure rate, \( \phi \).

Given the current \( z \) and \( \phi \), the worker’s utility flow per unit time employed is \( z - s(\phi) \), while his job-loss hazard rate is \( \alpha \phi \). Therefore, if \( r \) is the worker’s discount rate, the expected present value of the worker’s utility over the life of his job, is given by

\[
u(\phi | z, \alpha) = \frac{z - s(\phi)}{r + \alpha \phi}, \tag{1}
\]

and the first-order condition for maximising \( u(\phi | z, \alpha) \) is

\[
\alpha[z - s(\phi)] = -(r + \alpha \phi) s'(\phi). \tag{2}
\]

The properties of \( s \) (see Appendix) guarantee that \( (2) \) has a unique solution for \( \phi \) that maximises \( u(\phi | z, \alpha) \). We have:

**Proposition 1.** Let \( \hat{\Phi}(z, \alpha) \) be defined as the solution of \( (2) \) for \( \phi \). Then, \( \hat{\Phi} \) is a strictly dominant strategy for every worker, which yields him strictly positive rents. Furthermore, the worker’s optimal failure rate \( \hat{\Phi}(z, \alpha) \) is strictly decreasing in \( z \) and \( \alpha \).

**Proof.** Strict dominance follows from the fact \( \hat{\Phi}(z, \alpha) \) forms the worker’s unique best response to \((z, \alpha)\) whatever the previous history of play. We know that worker rents are strictly positive, because \( u(\phi | z, \alpha) \) is positive for sufficiently large \( \phi \). Implicit differentiation of \( (2) \) with respect to \( \alpha \) and to \( z \) yields

\[
\frac{\partial \hat{\Phi}}{\partial \alpha} = \frac{r}{\alpha (r + \alpha \phi)} s'(\phi) \tag{3}
\]

and

\[
\frac{\partial \hat{\Phi}}{\partial z} = -\frac{\alpha}{(r + \alpha \phi) s'(\phi)} \tag{4}
\]

Inasmuch as \( s' \) is negative and \( s'' \) is positive, we have immediately that both derivatives are everywhere negative.

Suppose, now, that all workers adopt their strictly dominant strategy, \( \hat{\Phi} \). Then, in any stage game, the employer will face the failure rate \( \hat{\Phi}(z, \alpha) \), where \( z \) and \( \alpha \) are the wage and firing probability in effect in that stage. The expected cost of a failure is \( L + \alpha F \), so that expected employer costs per unit time are given by

\[
b(z, \alpha | L, F) = z + \hat{\Phi}(z, \alpha) (L + \alpha F). \tag{5}
\]

For any fixed \( \alpha \), it is clear that \( b \to \infty \) both as \( z \to 0 \) and as \( z \to \infty \). Since \( b \) is strictly convex in \( z \), there must be a unique \( \hat{z}(\alpha) \) that minimises \( b(z, \alpha | L, F) \) with respect to \( z \), where \( \hat{z}(\alpha) \) is given by the solution of the first order condition

\[
\frac{\partial \hat{\Phi}}{\partial z} (L + \alpha F) = -1. \tag{6}
\]

Consequently, having chosen \( \alpha \), the employer in equilibrium must choose the
wage \( \bar{z}(\alpha) \). Defining \( \bar{z}_1 = \bar{z}(1) \) and \( \hat{\phi}_1 = \Phi(\bar{z}_1, 1) \), I demonstrate the following proposition:

**Proposition 2.** Suppose \( L \) is specified in the game with employer commitment. Then for \( F \) sufficiently small, the strategy profile \( \{ \bar{z}_1, 1 \}; \Phi \) is a unique subgame-perfect equilibrium. The equilibrium wage, \( \bar{z}_1 \), and the accrual rate of the worker’s equilibrium economic rent, \( \hat{\phi}_1 - s(\hat{\phi}_1) \), increase with \( L + F \).

**Proof.** To begin, note that (3) and the properties of \( s \) imply that \( \hat{\phi} \to \infty \) as \( \alpha \to 0 \). Thus, we can choose \( z, \bar{z}, \bar{a} > 0 \) to define a compact set \( A \equiv [z, \bar{z}] \times [\bar{a}, 1] \subset A \) such that \( \bar{b}(z, \alpha) \leq \bar{b}(1, 1) \) implies \( (z, \alpha) \in A \). It follows from the continuity of \( b \) on \( A \) that \( b \) takes a minimum on \( A \), that this minimum is the global minimum of \( b \) on all of \( A \), and that \( b \) never takes its minimum value outside of \( A \).

To prove the first assertion of the proposition, it remains to show only that \( \bar{z}_1 \) uniquely minimises \( b[\bar{z}(\alpha), \alpha \mid L, F] \) on the compact set \( A \). Applying the envelope theorem to (5), differentiating with respect to \( \alpha \), and using (3) and (6), yields

\[
\frac{db}{d\alpha} = \frac{rs'(\hat{\phi})}{\alpha^2} + \hat{\phi}F,
\]

where \( \hat{\phi} = \Phi[\bar{z}(\alpha), \alpha] \). The expression \( rs'(\hat{\phi})/\alpha^2 + \hat{\phi}F \) is everywhere negative in \( \alpha \), which we denote by \( b \). Then, for any \( F < |h|, db/d\alpha \) is everywhere negative in \( \alpha \). It follows immediately that \( (\bar{z}_1, 1) \) must be a unique best response to the workers’ dominant strategy \( \Phi \).

To show that \( \bar{z}_1 \) increases with \( L + F \), we apply the substitution indicated by (4) to (6) and set \( \alpha = 1 \), which yields the following implicit definition of \( \bar{z}_1 \):

\[
L + F \equiv \left[ r + \Phi(\bar{z}_1, 1) \right] s''[\Phi(\bar{z}_1, 1)].
\]

Treating \( \bar{z}_1 \) as a function of \( L + F \), we differentiate (8) implicitly, apply the substitution indicated by (4) (with \( \alpha = 1 \)), and then use the inequalities (A 2) and (A 4) to arrive at the desired result.

To demonstrate that economic rents increase with \( L + F \), we set \( \alpha = 1 \) in (2), which yields the identity

\[
\bar{z}_1 - s(\hat{\phi}_1) \equiv - (r + \hat{\phi}_1) s'(\hat{\phi}_1).
\]

Since \( \bar{z}_1 \) increases as \( L + F \) increases, it follows that \( \hat{\phi}_1 \) decreases. It remains to show only that the derivative of the right-hand side of (9) with respect to \( \hat{\phi}_1 \) is negative, but this follows directly from (A 2) and (A 3). \( \blacksquare \)

How much economic rent will the worker receive in equilibrium? From (2) and (6) we have

\[
\bar{z}_1 - s(\hat{\phi}_1) = \frac{1}{|\eta|} \hat{\phi}_1(L + F).
\]

In principle, this formula should enable us to use observable data to impute the value of worker rents. The elasticity \( \eta \) is a technical parameter that might be estimated by engineers and human resources experts. For workers of a given type in a given position, average realised losses per worker would be a good
estimator of expected losses per worker, \( \hat{\phi}_1(L + F) \). Note that jobs characterised by easily avoidable failures are not likely to yield high rents, because such failures will rarely materialise when behaviour is efficient. Rather, it is those jobs characterised by failures that are both difficult to prevent and have high associated costs that tend to yield high rents.

From the employer's perspective, worker rents can be viewed as a cost of failure abatement. But worker rents do not represent a social cost. Consequently, the level of worker effort aimed at failure prevention in equilibrium will be socially suboptimal. The proof of this is straightforward.

II. B. The Responsibility Game without Employer Commitment

Suppose now that the employer cannot commit in advance to a worker-dismissal policy. Instead, she is free to decide how to respond to a failure after the worker chooses his action. This behaviour can be modelled by adding an additional node to each stage game. The revised stage game is constructed as follows:

(i) The employer sets a wage rate \( z \).
(ii) The worker sets a failure rate \( \phi \).
(iii) The employer chooses a dismissal probability \( \alpha \in [\epsilon, 1] \).\(^4\)
(iv) Given the worker's choice of \( \phi \) and the employer's choice of \( \alpha \), nature determines the time that the worker is to be dismissed.

As before we will use the word ‘tough’ to describe employer strategies with \( \alpha = 1 \), and ‘lenient’ to describe strategies with \( \alpha < 1 \). Worker strategies will be functions of the current value of \( z \) and previous history. I show:

Proposition 3. Suppose that for \( L \) given, \( F \) is sufficiently small so that the tough strategy profile is the unique subgame-perfect equilibrium of the game with employer commitment. Then the employer would garner at least as much utility in that equilibrium as she would in any subgame-perfect equilibrium of the game without commitment.

Proof. We know from Proposition 1 that the strategy defined by the function \( \Phi(z, \alpha) \) is the worker's unique best response to dismissal probability \( \alpha \) in a game without commitment. Consequently in any stage game of a subgame-perfect equilibrium, employer actions \( (z, \alpha) \) would always be accompanied by the worker action \( \phi = \Phi(z, \alpha) \). The proposition then follows from the proof of the first assertion of Proposition 2. \( \blacksquare \)

Having established that the employer can do no better in the game without commitment than she can do in the game with commitment, we ask whether or not she can do as well. Let \( \langle z, \alpha \rangle \) denote the strategy in which the employer maintains the same wage \( z \) and dismissal probability \( \alpha \) in every stage game.

\(^4\) The positive lower bound on \( \alpha \) ensures that the worker has a positive probability of being fired after a failure. The fact that the worker's best response to a zero probability of job loss is not defined in this game makes this weak restriction on the model desirable. The reader may wish to interpret \( \epsilon \) as the probability that the employer will be forced to fire the worker by external regulations.

\(^5\) Because the employer does not observe the current value of \( \phi \), this move is simultaneous with the previous move from a game-theoretic point of view.
Now consider the rough strategy profile \(\langle \hat{z}_{1,1}; \hat{\Phi}_{1}\rangle\), which is analogous to the unique equilibrium of the game with employer commitment. The strategy \(\hat{\Phi}_1\) is the worker’s best response to the employer’s tough strategy, and the wage \(\hat{z}_1\) is the employer’s best response to \(\hat{\Phi}_1\). But because \(\hat{\Phi}_1\) is history independent, the current choice of \(z\) cannot affect future play, so that setting \(z = 1\) cannot be a best response of the employer. Rather, once the worker moves, the employer will always want to choose \(z = \epsilon\) (leniency) in order to save on worker-replacement costs, however small they may be.

Nevertheless, there are reasonable history-dependent strategies on the part of workers and the employer that will support a subgame-perfect equilibrium with the same play as the tough-strategy profile. To see this, first note that the ‘lenient’ strategy profile \(\langle \hat{z}(\epsilon), \epsilon; \hat{\Phi}_{1}\rangle\) is itself a subgame-perfect equilibrium. Thus, we can use trigger strategies in the usual way to construct a subgame-perfect equilibrium that is tough on the play of the game. For suppose all workers adopt a strategy \(\hat{\Phi}_{1,\epsilon}\) defined by \(\hat{\Phi}_{1,\epsilon}(z) = \hat{\Phi}_{1}(z)\) when the employer has no history of leniency and by \(\hat{\Phi}_{1,\epsilon}(z) = \Phi_{\epsilon}(z)\) when the employer has shown leniency in any past moves. This strategy would make sense for workers who believe ‘once lenient, always lenient’. And let the employer use the strategy \(\langle \hat{z}_{1,\epsilon}, \hat{z}_{1,\epsilon}\rangle\) defined by \(\hat{z}_{1,\epsilon} = \hat{z}_{1}\) and \(\hat{z}_{1,\epsilon} = 1\) if she has never before been lenient but by \(\hat{z}_{1,\epsilon} = \hat{z}(\epsilon)\) and \(\hat{z}_{1,\epsilon} = \epsilon\) if she has been lenient anytime in the past. We show

Proposition 4. Suppose the employer cannot credibly commit to a firing policy. Then, for any \(F\) and \(L\) and for \(\epsilon\) sufficiently small, the strategy profile \(\langle \hat{z}_{1,\epsilon}, \hat{z}_{1,\epsilon}; \hat{\Phi}_{1,\epsilon}\rangle\) is a subgame-perfect equilibrium whose play is everywhere tough. If \(F\) is sufficiently small, this equilibrium maximises the employer’s return.\(^6\)

Proof. The strategy profile \(\langle \hat{z}_{1,\epsilon}, \hat{z}_{1,\epsilon}; \hat{\Phi}_{1,\epsilon}\rangle\) will be a subgame perfect equilibrium if the employer is never motivated to deviate from her policy of ‘always be tough when there has been no previous leniency’. What does she have to gain and lose by deviating in this situation? If she deviates by being lenient in a given stage game, she saves \(F\) with probability \(1 - \epsilon\), but in every future period she earns the payoff of the lenient equilibrium rather than that of the tough strategy profile. As \(\epsilon\) gets small, the expected number of failures per unit time in the lenient equilibrium becomes increasingly large, so that employer’s payoff goes to \(-\infty\), and the first assertion is proved. The second assertion now follows immediately from Proposition 3.

\(^6\) I would like to thank a referee for suggesting this general version of the proportion.
explores internal promotion in the context of an efficiency-wage model. He shows that the prospect of a future promotion can serve as an alternative to the threat of dismissal as a method for eliminating shirking in conjunction with efficiency wages. MacLeod and Malcomson (1989) show that an appropriate job ladder can both elicit worker effort and sort workers of differing abilities. Fairburn and Malcomson (1995) examine the tradeoff between using promotions for the purpose of making efficient job assignments and using them for the purpose of rewarding good performance in lower-paying jobs.

Carmichael (1983) and Malcomson (1984) study the use of job ladders as labour-tournament-like mechanisms with fixed wage bills. The key to the efficacy of their mechanisms is that wage rates are associated with jobs rather than individuals, and the number of jobs at each wage rate is held fixed. Employers cannot save money by firing workers in highly paid jobs, because those jobs would then have to be staffed by other workers at the same high wage rates. The weakness of the Lazear mechanism is thus avoided. Prendergast (1993) constructs a different type of model in which all jobs need not be filled. He allows for the possibility of promotion into different jobs with different productivities and different contractually specified wages. In the Prendergast model, the worker has an incentive to invest in his own human capital because of a promised promotion to a high-paying job if he does so, and the employer has an incentive to keep her promise because the self-trained worker will be sufficiently more productive in the high-paid job to more than justify the pay differential. Fairburn and Malcomson (1994) make a related but more general point in that they allow for contract renegotiation.

The models developed in this section differs substantially from all of these. We analyse internal promotion as a motivating device for workers in responsible jobs, and we examine the relationship between the degree of responsibility and the effectiveness of the promotion strategy. In our models, the wage bill may or may not be fixed in advance. And the different jobs need not differ in productivity or in any other way. This is not a labour-tournament model, because there is no competition between workers for promotion. The employer uses promotion as a device to reduce the rents that the worker would otherwise collect. The worker pays in advance for his future high-wage job by working in the low-wage job for wages reduced from what he would otherwise receive. And when the employer is unable to commit to the future promotion and future wage level, reputational concerns can enforce the contract, much as in Bull (1987).

Our promotion model characterises a firm with two responsible positions that must be filled at every point in time. These positions are identical except possibly for the size of losses associated with failure. We explore a game between an employer and the sequence of workers who fill these positions. As before we use two game variants that differ with regard to the employer’s ability to make commitments pertaining to the policies she adopts. In either case, the employer can choose to manage these two positions independently, setting an optimal efficiency wage for each in accordance with the analysis of Section II. Or she may choose a promotion strategy, in which case she changes the way that she
fills vacancies. Instead of hiring workers for both jobs from outside the firm, she fills the higher paying position by promoting the worker in the lower paying position.

When using the promotion strategy, the employer will choose to increase the efficiency wage of the high-paying job and decrease that of the low-paying job from what she would pay without promotion. Even if the two jobs are associated with potential losses of the same size, the employer will choose to pay a higher wage in one of the jobs than in the other and to promote from the low-paying to high-paying position. Thus, in the responsible-job environment, a job ladder may be structured purely for incentive reasons.

III.A. The Promotion Game with Employer Commitment

Many large and medium firms have fixed job-classification schemes. Rates of pay for different jobs within such a scheme tend to be stable: it is the individual’s path up the promotion ladder that is subject to a relatively high level of uncertainty (see, for example, Lawler (1990)). This promotion game is intended to model the optimal design of such a scheme, under the strong assumption that workers will regard the pay-scale as permanently fixed. This game also requires the employer to commit to this promotion policy in advance, and to simplify our calculations we assume that she fires a worker when (and only when) a failure occurs (i.e. \( \alpha = 1 \)). For this case, without loss of generality, we assume that replacement costs \( F \) are zero. Later in this section we will generalise the game in order to eliminate these restrictions.

Suppose now that a firm offers two jobs requiring identical skills, and suppose that the potential losses associated with the jobs are \( L_1 \) and \( L_2 \), respectively, with \( L_1 \leq L_2 \). Assume that both jobs have the same associated required-effort function \( s \), and that identical workers with identical utility functions are hired. We permit the employer to choose from two different regimes with regard to filling these positions:

\[ X. \] External hiring; no promotions. If either worker is fired, a new worker will be hired from outside the firm to take his place.

\[ Y. \] Internal promotion from Job 1 to Job 2. If the worker in Job 2 is fired, the incumbent in Job 1 is promoted into Job 2. If Job 1 becomes vacant, either through a promotion or a dismissal, a new worker will be hired from outside the firm to take his place.

The promotion game is in extensive form. There is an initial stage in which the employer sets wages \( z_1 \) and \( z_2 \) for the two responsible positions and chooses one of the promotion regimes, \( X \) or \( Y \). At this point, or whenever a worker is fired thereafter, a stage game with the following structure begins:

(i) Job vacancies are filled according to the regime chosen by the employer.
(ii) Worker 1 sets his failure rate \( \phi_1 \) [and his effort level \( s(\phi_1) \)].
(iii) Worker 2 sets his failure rate \( \phi_2 \) [and his effort level \( s(\phi_2) \)].
(iv) Nature selects the time of the next failure and the identity of the worker who fails.
We have:

The time of failure for each worker is independently distributed according to his own chosen failure rate as in the previous section, and distribution of the time of the first failure and identity of the worker who fails can be derived accordingly. A new stage game begins at the time of the first failure.

The employer’s strategy space consist of vectors of the form \((z_1, z_2, R)\). The employer is required to choose \(z_1\) and \(z_2\) sufficiently large so that neither of her workers prefers a non-responsible job. This implies that when \(R = X\), both \(z_1\) and \(z_2\) are non-negative; when \(R = Y\), \(z_1\) may be negative to the extent that the expected value of future promotions permits.

The strategy space of Worker 2 consists of functions \(\Phi^*_2\) that map the wages, \(z_1\) and \(z_2\), the promotion policy \(R\), the failure rate \(\phi_1\) chosen by Worker 1, and the history of previous stage games to a failure rate \(\phi_2 > 0\). The strategy space of Worker 1 consists of function pairs \((\Phi^*_1, \Phi^*_2)\), where \(\Phi^*_2\) is a Worker-2-type strategy to be applied in the event of promotion, and \(\Phi^*_1\), to be applied otherwise, maps \(z_1\) and \(z_2\), the promotion policy \(R\), and the history of previous stage games to a failure rate \(\phi_1 > 0\). A worker’s utility flow per unit time in Job \(i\) is given by \(z_i - s_i(\phi_i)\), and \(u_i\) denotes the expected present value of the future utility of a worker in Job \(i\).

Below, we demonstrate that there is a unique subgame-perfect equilibrium characterised by internal promotion and by a strictly higher wage for Job 2 than for Job 1, even when the levels of responsibility in the two jobs are the same.

(i) Worker Optimisation. Worker 2 faces a situation identical to that faced by the worker of Section II. Therefore, the strategy \(\Phi^*_2\) where \(\Phi^*_2(z_2) = \Phi(z_2)\) as defined by (2), is a strictly dominant strategy for Worker 2. It is not surprising that Worker 2’s move depends only on \(z_2\).

Once Worker 1 has been promoted to Job 2, he will face a situation identical to that of Worker 2, and he will have an identical dominant strategy. The same logic applies to Worker 1 at any time when the employer has chosen Regime \(X\).

It remains to derive Worker 1’s best response before promotion when the employer has chosen Regime \(Y\) and when Worker 2 is using his dominant strategy. By calculating the expected utility of Worker 1 conditional on an assumed vacancy in Job 2 at time \(\tau\), and then using the distribution of \(\tau\) implied by the failure rate \(\phi_2 = \Phi^*_2(z_2)\), we find that the first-order condition for maximising \(u_1\) is

\[
(r + \phi_2) [z_1 - s_1(\phi_1)] + \phi_2[z_2 - s_2(\phi_2)] = -(r + \phi_2)(r + \phi_1 + \phi_2)s_2'(\phi_1). 
\]

The properties of \(s\) (see Appendix) imply that (11) has a unique solution for \(\phi_1\) that maximises \(u_1\). By differentiating (11), we can show that the optimal \(\phi_1\) decreases when either \(z_1\) or \(z_2\) increases. The latter relationship is not obvious, because although the value of a promotion rises as \(z_2\) increases, Worker 2 will be motivated to be more careful and is less likely to lose his job to Worker 1. We have:

**Proposition 5.** Let the function \(\phi_1 = \Phi^*_1(z_1, z_2, R)\) be given by \(\Phi(z_1)\) when \(R = X\) and by the solution of (11) for \(\phi_1\) when \(R = Y\). Then, \(\Phi^*_1\) is the unique best response to
any employer strategy and the dominant strategy of Worker 2. Furthermore, $\Phi^*_1$ is increasing in both $z_1$ and $z_2$.

(ii) \textbf{Employer Optimisation.} We now characterise the employer’s cost-minimising efficiency wage and promotion regime, given the workers’ strategies $\Phi^*_1$ and $\Phi^*_2$. As in Section II, the employer’s failure-associated costs per unit time are constant: the sum of the efficiency wages and the expected losses from failures. Thus, the expected value of these costs is given by

$$c(z_1, z_2, R|L_1, L_2) = z_1 + \phi_1 L_1 + z_2 + \phi_2 L_2,$$

where $\phi_1 = \Phi^*_1(z_1, z_2, R)$ and $\phi_2 = \Phi(z_2)$. We can show that the employer strictly prefers the internal-promotion regime, $Y$, by demonstrating the following proposition:

\textbf{Proposition 6.} Given that $L_2 \geq L_1$, then in the case where the worker response functions are as defined in Proposition 5, we have:

$$\min_{z_1, z_2} c(z_1, z_2, Y|L_1, L_2) < \min_{z_1, z_2} c(z_1, z_2, X|L_1, L_2).$$

\textbf{Outline of Proof.} The analysis of Section II applies unchanged to each worker when the employer chooses Regime $X$. Optimal wage rates, $\hat{z}_1$ and $\hat{z}_2$ are each determined by (6) with $L_1$ and $L_2$ respectively substituted for $L$. At these wage rates, the workers’ choices of failure rates will be $\phi_1 = \Phi(\hat{z}_1)$ and $\phi_2 = \Phi(\hat{z}_2)$.

Let $c_Y(z_1, z_2|L_1, L_2)$ denote $c(z_1, z_2, X|L_1, L_2)$ and $c_Y(z_1, z_2|L_1, L_2)$ denote $c(z_1, z_2, Y|L_1, L_2)$. Because the function $c_Y$ takes a minimum, the left-hand side of the inequality in Proposition 6 is well defined. We argue that

$$c_Y(\hat{z}_1, \hat{z}_2|L_1, L_2) \leq c_Y(\hat{z}_1, \hat{z}_2|L_1, L_2).$$

The right-hand term denotes total cost when the employer has optimised efficiency wages for the external-hiring regime, $X$. The left-hand term denotes total cost if she then switches to the internal-promotion regime, $Y$, without modifying those efficiency-wage payments. Worker 2 would be entirely unaffected by the change, but Worker 1 would have the prospect of possible promotion to a job with an equal or increased salary and utility, and he would thus work at least as hard. It follows that total costs to the employer must remain the same or decrease.

Let $z^*_1$ and $z^*_2$ denote the values of $z_1$ and $z_2$ that minimise $c_Y$. That $c_Y(z^*_1, z^*_2|L_1, L_2)$ is strictly less than $c_Y(\hat{z}_1, \hat{z}_2|L_1, L_2)$ follows from the fact that

$$\frac{\partial c_Y}{\partial z_2}\hat{z}_1, \hat{z}_2 \neq 0,$$

which is demonstrated in the proof of Proposition 8, below.

Propositions 5 and 6 imply:

\textbf{Proposition 7.} Given $L_1 \leq L_2$, the strategies $\Phi^*_1$ and $\Phi^*_2$ for the workers and the strategy $(z^*_1, z^*_2, Y)$, for the employer, constitute a subgame-perfect equilibrium for the two-stage game. Faced with $\Phi^*_1$ and $\Phi^*_2$, the employer strictly prefers Regime $Y$ to Regime $X$.

We now explain why Regime $Y$ leads to wages that are further apart than those of Regime $X$.
Table 1

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<th>( \eta )</th>
<th>( \phi_1 )</th>
<th>( \phi_2 )</th>
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<th>( z_2^* )</th>
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<th>( U_2^* )</th>
<th>( C_1^* )</th>
<th>( C_2^*/C_X )</th>
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Note: \( \eta = -n, \xi \equiv \xi_1 = \xi_2, \phi = \Phi(\xi), U = U(\phi(\xi), \phi^* = \Phi^* \xi_1, \xi_2, Y), \phi^*_1 = \Phi^*_1(z_1^*, \xi_2, Y) \).
possibility of promotion. Because of the promotion possibility, expected rents in Job 1 are positive and thus there will be an excess supply of workers to Job 1 in Regime Y. Of course, this is true of both jobs in Regime X.

III.B. A Promotion Game without Employer Commitment

As noted above, promotion strategies can be made incentive-compatible as long as the employer can commit to a fixed number of positions at specified wages. We shall see that the incentive-compatibility of promotion strategies can be carried over even to games in which the employer cannot commit to a fixed number of positions at specified wages. In the equilibria of the games in this section, the payment of a high wage in the senior job is enforced by the strategy of the more junior worker, whose effort is elicited by the prospect of promotion. Any reduction of the high wage paid to the senior worker would be an unambiguous signal to the junior worker that the employer is reneging. Thus, the employer cannot profitably reduce the high wage paid in senior position, and it follows that she has no incentive to fire the senior incumbent in the absence of a job failure. Indeed, for the sake of simplicity, these games do not allow for the possibility of firing a worker unless a failure has occurred.

Consider, then, a version of the promotion game that allows the employer to set wage rates at the time workers are hired or promoted and allows her to decide on dismissal and promotion policies after worker strategies are chosen in a manner parallel to that of Section II.B. In this game the employer can choose to be tough or lenient \((\alpha \in [\epsilon, 1])\), and we assume that there is an explicit worker replacement cost \(F > 0\) when a worker is dismissed.

Now consider the strategy profile formed by the worker strategies \(\Phi_1^w\) and \(\Phi_2^w\) as defined in the previous section, and by the employer strategy with wages \(z_1^s\) and \(z_2^s\), internal promotion, and the probability of dismissal set equal to 1 for every stage game. In the promotion game in which employers can commit to future wage and worker-dismissal policies, these strategies form a subgame-perfect equilibrium; here they do not, and for two different reasons. First, when strategies are history independent and replacement costs are positive, choosing toughness after the fact cannot be an employer’s best response. Secondly, and more importantly, Worker 1 is not at all interested in the current level of \(z_2^s\), but rather in the level that would prevail in the event of his promotion. So if the employer, having adopted \(z_2^s\) for all future periods, deviates from \(z_2^s\) in the current period, the behaviour of Worker 1 would be unaffected. But this makes \(z_2^s\) not \(z_2^s\), the employer’s best response to \(\Phi_1^w\) and \(\Phi_2^w\) in the first stage of any given subgame.

Yet, despite these problems with history-independent strategies, we can readily define trigger strategies for this game to support the equilibrium play of the promotion game with commitment, as described in Proposition 7. The "bad" equilibrium used for enforcement is characterised by employer leniency \((\alpha = \epsilon)\), external promotion, worker strategies of the form \(\hat{\Phi}_2\), and wage rates \(\hat{z}(\epsilon)\), all as defined in Section II.B. An argument that parallels the proof of Proposition 4 yields:

*I would like to thank a referee for clarifying this issue for me.*
Proposition 9. Suppose the employer cannot credibly commit to her wage rate and her firing and promotion policies. Then, for sufficiently small worker replacement costs \( F \), the trigger strategies defined above form a subgame-perfect equilibrium whose play is everywhere tough.

IV. CONCLUSION

I have considered the nature of responsible jobs in an environment with limited contract enforceability. I modelled a scenario in which job failures cause the employer to sustain large losses, and in which increased worker effort lowers the failure rate. I measured the degree of job responsibility by the size of the losses associated with failures. In order to induce the worker to offer an appropriate level of effort, the employer pays an above-market wage and threatens to dismiss the worker if a failure occurs.

In this abstract setting, there are self-enforcing contracts in which profit-maximising employers pay workers sufficiently well so as to provide them with economic rents. The value of these rents, in aggregate, approximates a well-defined fraction of aggregate realised losses caused by work failures. This fraction is determined by the technological relationship between effort and the failure rate. These results imply a need for empirical investigations of that relationship; I know of no such studies in the literature.

Employers have to pay rents to workers, in our setting, because they cannot enforce contracts that would charge workers for losses after they occur. Furthermore, because of potential moral hazard, employers cannot recover those rents by requiring potential employees to pay for their jobs in advance. However, employers that offer several responsible jobs can partially recover the rents they pay: they can induce workers in low-paying jobs to offer increased effort as a way of paying for promotions to high-paying jobs. Internal promotion saves employers money because they enjoy a kind of double counting in the incentive effect of increased rents in the high-paying jobs. Workers in the high-paying jobs work harder because of increased rents; workers in less well-paying jobs also work harder, because they have the prospect of being promoted to a more valuable job later.

I have demonstrated that in the model, it is in the interest of the employer to construct an internal job ladder: i.e. to hire workers for the lowest-paying jobs from outside the firm and to promote workers to higher-paying jobs from within the firm. With a job ladder, the employer will set wage levels that are more disparate than they would be if all jobs were filled from outside the firm. The most surprising result of this paper suggests that even when jobs and responsibilities are identical, it is profit maximising to create a job-ladder and offer different wages to different workers. To those of us in academic life this is indeed a familiar scenario.

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APPENDIX: RESTRICTIONS ON THE FUNCTION $s(\phi)$

Let $\eta_s$ and $\eta_r$ denote the elasticities of $s$ and $r$ with respect to $\phi$, and let $\eta_{sr}$ and $\eta_{rs}$ be elasticities of these elasticities. We assume that $s(\phi)$ satisfies the following regularity conditions:

(i) $s$ is continuous and three-times differentiable
(ii) $s > 0$ and $s' < 0$;
(iii) $s \to \infty$ as $\phi \to 0$ and $s \to 0$ as $\phi \to \infty$;
(iv) $\eta_s$ and $\eta_r$ are bounded on the domain of $s(\phi)$, $\eta_{sr} \leq |\eta_s|$, and $\eta_{rs} \leq |\eta_r|$.

Condition (iv) constrains how much and the rate at which the elasticities $\eta_s$ and $\eta_r$ may vary, and thereby ensures the convexity of the employer's cost function and the monotonicity of optimal moves as functions of model parameters. The set of effort functions satisfying all of these conditions is a broad one; in particular, all of the constant-elasticity functions, $s(\phi) = A/\phi^a$, with $A, a > 0$, are included.

Conditions (i)–(iv) imply that $s$ has several other properties useful for demonstrating the uniqueness of optima and monotonicity in comparative statics. The regularity conditions, $\eta_{sr} \leq |\eta_s|$, and $\eta_{rs} \leq |\eta_r|$, and the mathematical identity, $\eta_r \equiv \eta_s - 1 + \eta_{sr}$, imply that

$$\eta_r \leq -1 \quad \text{and} \quad \eta_{sr} \leq -1,$$

so that, since $s' < 0$, we have

$$s'' > 0 \quad \text{and} \quad s''' < 0.$$

It follows from (A 1) and (A 2) that

$$\frac{d}{d\phi} \phi s'(\phi) \equiv \phi s'' + s' \geq 0$$

(A 3)

and

$$\frac{d}{d\phi} \phi s''(\phi) \equiv \phi s''' + s'' \leq 0.$$

(A 4)

REFERENCES


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