Relational Thinking

(Adapted from Stump, Bishop, & Roebuck)

Consider the following number sentences. The goal is *not* to compute the sums, differences, products, or quotients. Instead, the purpose is to analyze the relationships among the numbers. Without calculating, determine whether each number sentence is true or false and explain your reasoning.

1. $53 + 86 = 51 + 88$
2. $65 + 41 = 68 + 44$
3. $95 – 52 = 93 – 54$
4. $76 – 37 = 78 – 39$
5. $53 \times 34 = 51 \times 36$
6. $48 \times 36 = 16 \times 72$
7. $48 \times 36 = 24 \times 72$
8. $16 \times 27 = 32 \times 54$
9. $42 \div 14 = 21 \div 7$
10. $24 \div 8 = 48 \div 4$

Discuss: Problems like these challenge children, and adults, too, to think about and describe the effect of mathematical operations on relationships among quantities.

Separate the sentences into those that are true and those that are not true. Describe the types of changes that produced true sentences. Did these changes differ for different operations?

Study the true number sentences above. Then for each unfinished number sentence below, think about ways you could change the original pair of numbers to create a new pair of numbers that makes the equation a true statement.

1. $75 + 43 =$ _____ + _____
2. $165 – 97 =$ _____ – _____

Relational thinking can be sued to solve equations as well as to determine whether or not they are true. Use relationships like those you identified in the first part of this activity to solve the following equations *without calculating*. For each equation, explain the relationship you used to solve the equation.

1. $153 + 572 = 149 + \square$
2. $24 \times 42 = 48 \times \square$
3. $\square \div 9 = 360 \div 45$
4. $172 - \square = 275 - 182$
5. $842 - 721 = \square - 735$
6. $144 \times \square = 20 \times 288$

Reflect: These questions will help you summarize what you have learned.

1. How do you solve an equation that has similar addends on both sides?

2. How do you solve an equation that has similar factors on both sides?

3. How do you solve an equation that involves subtraction on both sides?

4. How do you solve an equation that involves division on both sides?

Think about the answers to these questions, discuss your ideas with others, and then write a summary of what you learned from this activity.