Assignment #1

4a) \( PV = nRT \) so \( P = \frac{nRT}{V} \)

\[
\frac{\partial P}{\partial V} = \frac{nRT}{V^2} \left( -V^{-2} \right) = -\frac{nRT}{V^2}
\]

b) \( r = \sqrt{x^2 + y^2 + z^2} = (x^2 + y^2 + z^2)^{1/2} = q \sqrt{x^2 + y^2 + z^2} \)

Chain rule:

\[
\frac{\partial r}{\partial x} = \frac{\partial r}{\partial q} \cdot \frac{\partial q}{\partial x} = \frac{1}{2} q^{-1/2} \left( 2x \right) \]

so

\[
\frac{\partial r}{\partial x} = \frac{q^{-1/2} y}{\sqrt{x^2 + y^2 + z^2}}
\]

2) \( T = 273K \) \( B = -21.7 \text{ cm}^3 \text{ mol}^{-1} \)

\( C = 1.200 \times 10^3 \text{ cm}^3 \text{ mol}^{-1} \) \( \rho = 1.00 \times 10^3 \text{ kg m}^{-3} \)

\[
\begin{align*}
Z &= 1 + \frac{B}{V_n} + \frac{C}{V_n^2} + \ldots \\
\text{from } PV &= nRT \Rightarrow V_n &= \frac{RT}{P} \\
&= \frac{(0.3145 \text{ atm})(273K)}{(100 \times 10^3 \text{ atm})(0.1875 \text{ Pa atm})} = 224.6 \text{ cm}^3 \text{ mol}^{-1}
\end{align*}
\]

Recall \( V_n \) is the molar volume.

\[
Z = 1 + \frac{-21.7 \text{ cm}^3 \text{ mol}^{-1}}{224.6 \text{ cm}^3 \text{ mol}^{-1}} = 0.927
\]

\[
Z = PV_n \Rightarrow V_n = \frac{RT}{P} = \frac{(0.927)(0.05206 \text{ L atm} \text{ mol}^{-1} \text{ K}^{-1})(273K)}{1.00 \times 10^3 \text{ atm}} = \frac{0.208 \text{ L}}{\text{ mol}}
\]

\[
(2.08 \text{ cm}^3 \text{ mol}^{-1})
\]
3) O₂ gas  \[ V = 1.50 \text{ L} \]  \[ P = 168 \text{ kPa} \]  \[ T_{all} = 25.00^\circ \text{C} = 298.15 \text{ K} \]

\[ a) \sqrt{\frac{V}{M}} = \sqrt{\frac{2RT}{M}} \]
\[ M = 2\left(15.999 \text{ g mol}^{-1}\right) \left(0.031998 \text{ g} \text{ mol}^{-1}\right) = 0.031998 \text{ g} \text{ mol}^{-1} \]
\[ \sqrt{\frac{V}{M}} = \left(\frac{3\left(8.314 \text{ J mol}^{-1} \text{ K}^{-1}\right)(298.15 \text{ K})}{0.031998 \text{ g} \text{ mol}^{-1}}\right)^{1/2} = 682.89 \text{ J} \text{ mol}^{-1} \]

\[ b) \dot{E}_k = \frac{3}{2} k \bar{e} = \frac{3}{2} \left(1.3806 \times 10^{-23} \text{ J K}^{-1}\right)(298.15 \text{ K}) = 6.766 \times 10^{-21} \text{ J} \]
\[ \text{ (per molecule)} \]

\[ c) \dot{E}_k = \dot{E}_k \cdot \frac{N}{n_\text{mol mole}} \]

\[ PV = nRT \quad \Rightarrow \quad n = \frac{PV}{RT} \quad \text{and} \quad N = n \cdot \frac{V}{RT} \]
\[ N = \frac{(168 \times 10^3 \text{ Pa})(1.50 \times 10^{-3} \text{ m}^3)(6.022 \times 10^{23} \text{ mol}^{-1})(8.314 \text{ J mol}^{-1} \text{ K}^{-1})(298.15 \text{ K})}{(0.031998 \text{ g} \text{ mol}^{-1})} = 6.122 \times 10^{22} \text{ molecules} \]

\[ \dot{E}_k = \frac{(6.1744 \times 10^{-21} \text{ J})}{(6.122 \times 10^{22} \text{ molecule})} = 378 \text{ J} \]
b) Mean free path is given by equation (1.68)

\[ \lambda = \frac{\sqrt{2m \cdot kT}}{\pi a^2 N_a} \]

If you decrease the container size, you decrease the vol. V. If V ↓ then, \( \lambda \) ↓ so the mean free path will be lower (less dist travelled between collisions).

That makes sense since since the particles have less volume to move around in, the chance of collision is greater so they will travel on average, less between collisions.

c) We know that the compressibility factor \( Z \) is defined in equation (1.98)

\[ Z = \frac{PV}{RT} \]

If you look at Fig 1.17, it’s clear that as the P ↓ \( Z \) approaches zero, the value of an real gaseous approaches ideal. If \( Z = 1 \). If the gas is ideal it has \( Z = 1 \) at all P.

Real gaseous will approach ideal behavior at very low P.

(Where the assumptions we make about ideal gases (page 17) are most closely true)
1) From the ideal gas equation we can see that as \( T \) (if \( v \) is constant) then \( P \) must also increase.