1) The force constant \( k \) of the \(^{35}\text{Cl}^{35}\text{Cl} \) molecule is 320 N m\(^{-1} \). Calculate the fundamental vibration frequency (in cm\(^{-1} \) and s\(^{-1} \)) and the zero point energy (in J) of this molecule.

2) Construct the full wavefunction (including the normalization constant) for the harmonic oscillator in the \( \nu = 4 \) state.

3) Show explicitly that for a harmonic oscillator in the \( \nu = 2 \) state, the expectation value \( \langle x^2 \rangle = \int_{-\infty}^{\infty} \psi_2^* x^2 \psi_2 \, dx = \frac{5}{2} \frac{\hbar}{\sqrt{\mu k}} \). You will need to set up the \( \psi_2 \) wavefunction and evaluate the appropriate integral. Is this consistent with what you obtain from the general formula in equation (5-53)? Recall that \( \alpha = \left( \frac{\mu k}{\hbar^2} \right)^{1/2} \)

4) Prove explicitly that the \( \psi_0 \) and \( \psi_2 \) harmonic oscillator wavefunctions are orthogonal. You will need to use some of the standard integrals on the front page of McQuarrie. Please make sure your proof is clear and you explain what you are doing at each step. **Hint:** there is no need to explicitly include the normalization constants in your solutions – you can refer to the normalization constants for the \( \nu = 0 \) and \( \nu = 2 \) states as \( N_0 \) and \( N_2 \) respectively.

5) A real diatomic molecule has a potential energy curve that deviates dramatically from a harmonic oscillator model as the displacements about the equilibrium bond length increase. One way we can approximate the true behavior of molecules is via a Morse potential which has the form \( U(x) = D_e \left( 1 - e^{-\beta x} \right)^2 \) where \( x \) is the displacement from the equilibrium bond length.

For HCl \( D_e \) has the value \( 7.31 \times 10^{-19} \) J / molecule and \( \beta = 1.82 \times 10^{10} \) m\(^{-1} \).

(a) Plot (using Excel) the form of this Morse potential for the HCl molecule (for displacements between \(-0.5 \) Å and \(+3.0 \) Å). Be careful with the units of your potential energy.

(b) Given that the potential may be expanded in a Taylor series about \( x = 0 \) to give \( U(x) = D_e \beta^2 x^2 - D_e \beta^3 x^3 + \cdots \) calculate the force constant for HCl, explaining your reasoning. Then use this value of \( k \) to plot a harmonic oscillator potential (on the same plot as your Morse potential) for displacements between \(-0.7 \) Å and \(+0.7 \) Å. (The different ranges for the two potentials have been chosen to allow both potentials (which have slightly different energy scales) to fit easily onto one plot). Notice how the two plots (Morse and HO potentials) differ. Include this plot with your assignment. If you have difficulty with Excel, please seek help.