1) The lowest frequency pure rotational transition for the $^{12}\text{C}^{32}\text{S}$ diatomic molecule is found at 48991.0 MHz. Calculate the C–S bond length (in Å) in this species. Mass data: $m(^{12}\text{C}) = 12.000000$ amu, $m(^{32}\text{S}) = 31.97207$ amu. Be careful with sig figs! 1 amu = $1.66056 \times 10^{-27}$ kg.

2) The Laplacian operator for a rotor in 2D, in Cartesian coordinates $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ may be written in the more useful polar coordinates using either of the following two forms:

$$\left( \nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \quad \text{or} \quad \left( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right)$$

Prove that the radial portions of the two forms of $\nabla^2$ are indeed equivalent (the radial terms – that is, the terms which have an $r$ dependence but no $\phi$ dependence – are circled). (Note: Problems 6-10 and 6-11 in McQuarrie actually derive the polar form of the Cartesian form of the Laplacian by repeated application of the chain rule – you should take a look at this and see how the forms given above are derived. Problem 6-10 does the same for a fixed value of $r$ (and so, since $r$ is a constant, the derivatives with respect to $r$ will disappear) while Problem 6-11 does it for a case where $r$ is also variable to obtain the forms given above).

3) A rigid rotor is in a state whose eigenfunction is $Y_4^3(\theta, \phi)$ (where we represent the eigenfunction by a spherical harmonic of the form $Y_l^m(\theta, \phi)$). (Big) hint: you don’t need to construct the actual wavefunction to answer this question.

(a) What is the rotational energy of the rotor? (Give your answer in terms of the moment of inertia ($I$) and $\hbar$).

(b) What is the magnitude of the total angular momentum of the rotor?

(c) What is the magnitude of the $z$-component of the angular momentum (i.e. what is the magnitude of $L_z$ in units of $\hbar$)?

4) Does the $Y_4^3(\theta, \phi)$ eigenfunction exist for the rigid rotor? Explain your answer.

5) Since the $\hat{L}^2$ operator and the $\hat{L}_z$ operator commute (see Example 6-8 in McQuarrie) we can precisely (and simultaneously) measure values of the total angular momentum squared ($L^2$) and the $z$-component of the angular momentum ($L_z$).

Show whether the $\hat{L}_x$ operator and the $\hat{L}_y$ operator commute (in other words, evaluate the commutator $[\hat{L}_x, \hat{L}_y]$). What consequences does this have on our ability to measure simultaneously the values of the $x$- and $y$- components ($L_x$ and $L_y$) of the angular momentum? Use the forms of the operators in Cartesian coordinates as given in equation (6-84) on page 217 of McQuarrie.