1) The potential \((U(x))\) of an oscillator may be given by the expression
\[
U(x) = \frac{k}{2} x^2 + \frac{\gamma}{6} x^3 + \frac{b}{24} x^4
\]
where \(x\) is the displacement from equilibrium and \(\gamma, b,\) and \(k\) are constants of the system.
(a) Identify the form of \(\hat{H}^{(0)}, \hat{H}^{(1)}\) and \(\psi^{(0)}\) (that is, the unperturbed Hamiltonian, the perturbation Hamiltonian and the ground state wave function) for this system.  
**Hint:** the harmonic oscillator (HO) model should be used as the unperturbed system.

(b) Using the HO model as the unperturbed system, calculate the first-order correction to the ground-state energy for this system.

2) Using a trial function of the form \(e^{-\alpha r^2}\) to describe the ground state of the hydrogen atom (with \(\alpha\) acting as a variational parameter), prove explicitly that
\[
E(\alpha) = \frac{3\hbar^2 \alpha}{2 \mu} - \frac{e^2 \alpha^{1/2}}{2^{1/2} \varepsilon_0 \pi^{1/2}}
\]
You will need to use the Hamiltonian given in equation (7-32) and evaluate equation (7-29) to get \(E(\alpha)\).  **Show your work clearly.** How would you determine the value of \(\alpha\) that minimizes the energy?  (You can just explain how to do this; you don’t need to actually carry out the calculations!)

3) We might use the trial function given below to carry out a variational calculation for the ground state of the hydrogen atom.
\[
\phi = c_1 e^{-\alpha r^2} + c_2 e^{-\beta r^2}
\]
**Without doing any calculations,** deduce what the values of \(c_1, c_2, \alpha, \beta\) and \(E_{\text{min}}\) will be.  **Explain your reasoning.**

4) Expand the determinant below and solve for \(x\).
\[
\begin{vmatrix} 2x & 4 \\ 2 & x \end{vmatrix} = 2
\]