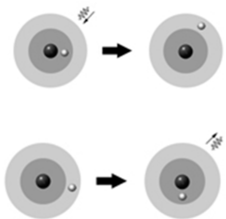
Physics 1161: Lecture 23

Models of the Atom

Sections 31-1 – 31-6



Bohr model works, approximately

Hydrogen-like energy levels (relative to a free electron that wanders off):

$$E_n = -\frac{mk^2e^4}{2\hbar^2}\frac{Z^2}{n^2} \approx -\frac{13.6 \cdot Z^2}{n^2}\ \text{eV}\ \left(\text{where } \hbar \equiv h/2\pi\right)$$

⇐ Energy of a Bohr orbit

Typical hydrogen-like radius (1 electron, Z protons):

$$r_n = \left(\frac{h}{2\pi}\right)^2 \frac{1}{mke^2}\frac{n^2}{Z} = \left(0.0529\ \text{nm}\right)n^2$$

⇐ Radius of a Bohr orbit

A single electron is orbiting around a nucleus with charge +3. What is its ground state (n=1) energy? (Recall for charge +1, E= -13.6 eV)

1) E = 9 (-13.6 eV)
2) E = 3 (-13.6 eV)
3) E = 1 (-13.6 eV)

Muon Checkpoint

$$r_n = (\frac{h}{2\pi})^2 \frac{1}{mke^2}\frac{n^2}{Z} = \underbrace{(0.0529nm)}_{\text{Bohr radius}}\frac{n^2}{Z}$$

If the electron in the hydrogen atom was 207 times heavier (a muon), the Bohr radius would be

1) 207 Times Larger 19 %
2) Same Size 53 %
3) 207 Times Smaller 28 %

(Z =1 for hydrogen)

Transitions + Energy Conservation

- Each orbit has a specific energy:

Photon emitted when electron jumps from high energy to low energy orbit. Photon absorbed when electron jumps from low energy to high energy:

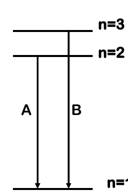
Photon Emission Checkpoint

Electron A falls from energy level n=2 to energy level n=1 (ground state), causing a photon to be emitted.

Electron B falls from energy level n=3 to energy level n=1 (ground state), causing a photon to be emitted.

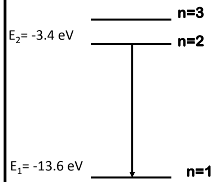
Which photon has more energy?

- **Photon A**
- **Photon B**



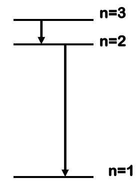
Example — Spectral Line Wavelengths

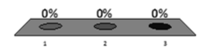
Calculate the wavelength of photon emitted when an electron in the hydrogen atom drops from the n=2 state to the ground state (n=1).



Compare the wavelength of a photon produced from a transition from n=3 to n=2 with that of a photon produced from a transition n=2 to n=1.

1. $\lambda_{32} < \lambda_{21}$
2. $\lambda_{32} = \lambda_{21}$
3. $\lambda_{32} > \lambda_{21}$

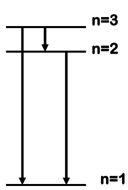




Photon Emission Checkpoint

The electrons in a large group of hydrogen atoms are excited to the n=3 level. How many spectral lines will be produced?

(1)	(2)	(3)
(4)	(5)	(6)



Bohr's Theory & Heisenberg Uncertainty Principle Checkpoints

So what keeps the electron from "sticking" to the nucleus?

Centripetal Acceleration
Pauli Exclusion Principle
Heisenberg Uncertainty Principle

To be consistent with the Heisenberg Uncertainty Principle, which of these properties can not be quantized (have the exact value known)? (more than one answer can be correct)

Electron Orbital Radius
Electron Energy
Electron Velocity
Electron Angular Momentum

Quantum Mechanics

- Predicts available energy states agreeing with Bohr.
- Don't have definite electron position, only a probability function.
- Orbitals can have 0 angular momentum!
- Each electron state labeled by 4 numbers:
  - n = principal quantum number (1, 2, 3, …)
  - ℓ = angular momentum (0, 1, 2, … n-1)
  - m_ℓ = component of l ($-\ell < m_\ell < \ell$)
  - m_s = spin (-½ , +½)

Quantum Numbers

Summary

- **Bohr's Model gives accurate values for electron energy levels…**
- **But Quantum Mechanics is needed to describe electrons in atom.**
- **Electrons jump between states by emitting or absorbing photons of the appropriate energy.**
- **Each state has specific energy and is labeled by 4 quantum numbers (next time).**

Bohr's Model

- Mini Universe
- Coulomb attraction produces centripetal acceleration.
  - This gives energy for each allowed radius.
- Spectra tells you which radii orbits are allowed.
  - Fits show this is equivalent to constraining angular momentum L = mvr = n h

Bohr's Derivation 1

Circular motion $\frac{mv^2}{r} = \frac{kZe^2}{r^2} \Longrightarrow \frac{1}{2}mv^2 = \frac{kZe^2}{2r}$

Total energy $E = \frac{1}{2}mv^2 - \frac{kZe^2}{r} = -\frac{kZe^2}{2r}$

Quantization of angular momentum: $(mvr)_n = mv_n r_n = n\frac{h}{2\pi}$

$\Longrightarrow v_n = n\frac{h}{2\pi m r_n}$

Bohr's Derivation 2

Use $v_n = n\frac{h}{2\pi m r_n}$ **in** $mv_n^2 = \frac{kZe^2}{r_n}$

$\Longrightarrow r_n = n^2\left(\frac{h}{2\pi}\right)^2\frac{1}{mkZe^2} = (0.0529nm)\frac{n^2}{Z}$

"Bohr radius"

Substitute for r_n in $E_n = -\frac{kZe^2}{2r_n}$

$\Longrightarrow E_n = -13.6eV\frac{Z^2}{n^2}$

Note:
r_n has Z
E_n has Z^2

Quantum Numbers

Each electron in an atom is labeled by 4 #'s

n = Principal Quantum Number (1, 2, 3, …)

- Determines energy

ℓ = Orbital Quantum Number (0, 1, 2, … n-1)

- Determines angular momentum

$L = \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$

m_ℓ = Magnetic Quantum Number (ℓ , … 0, … -ℓ)

- Component of ℓ

$L_z = m_\ell\frac{h}{2\pi}$

m_s = Spin Quantum Number (+½ , -½)

- "Up Spin" or "Down Spin"

Nomenclature

"Shells"	"Subshells"
n=1 is "K shell"	ℓ =0 is "s state"
n=2 is "L shell"	ℓ =1 is "p state"
n=3 is "M shell"	ℓ =2 is "d state"
n=4 is "N shell"	ℓ =3 is "f state"
n=5 is "O shell"	ℓ =4 is "g state"

Example

1 electron in ground state of Hydrogen:

n=1, ℓ =0 is denoted as: $1s^1$

n=1 · ℓ=0 · 1 electron

Example: Quantum Numbers

How many unique electron states exist with n=2?

ℓ = 0 : $2s^2$

$m_\ell = 0$: m_s = ½ , -½ — 2 states

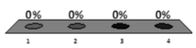
ℓ = 1 : $2p^6$

m_ℓ = +1: m_s = ½ , -½ — 2 states
m_ℓ = 0: m_s = ½ , -½ — 2 states
m_ℓ = -1: m_s = ½ , -½ — 2 states

There are a total of 8 states with n=2

How many unique electron states exist with n=5 and m_ℓ = +3?

1. 2
2. 3
3. 4
4. 5



Pauli Exclusion Principle

In an atom with many electrons only one electron is allowed in each quantum state (n, ℓ, m_ℓ, m_s).

This explains the periodic table!

What is the maximum number of electrons that can exist in the 5g (n=5, ℓ **= 4) subshell of an atom?**

Electron Configurations

Atom	Configuration		
H	$1s^1$		
He	$1s^2$	1s shell filled	(n=1 shell filled - noble gas)
Li	$1s^2 2s^1$		
Be	$1s^2 2s^2$	2s shell filled	
B	$1s^2 2s^2 2p^1$		
etc			
Ne	$1s^2 2s^2 2p^6$	2p shell filled	(n=2 shell filled - noble gas)

s shells hold up to 2 electrons

p shells hold up to 6 electrons

Sequence of Shells

Sequence of shells: 1s,2s,2p,3s,3p,4s,3d,4p…..

4s electrons get closer to nucleus than 3d

19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn						
4s		3d										4p					

In 3d shell we are putting electrons into ℓ = 2; all atoms in middle are strongly magnetic.

Angular momentum ⟹ **Loop of current** ⟹ Large magnetic moment

Example

Sodium

Na $1s^2 2s^2 2p^6\ 3s^1$ ← Single outer electron

Neon - like core

Many spectral lines of Na are outer electron making transitions

Yellow line of Na flame test is
3p ⟶ 3s

www.WebElements.com