MAT 53353: ADVANCED PERSPECTIVES ON THE CALCULUS HOMEWORK 1

Exercise 1.1. Give, in your own words, a definition for the limit of a function. You are not, however, to use any words that imply the existence or passage of time nor any reference to motion (which, after all, requires time). For example, "as x goes to..." is right out.

Exercise 1.2. What do we mean by "a well-defined function"? Give an example of a "function" that is not well-defined.

Exercise 1.3. David Bressoud¹ has voiced the opinion that limits, as a subject, have no place in a first-semester calculus course. In the context of our discussion, is this in keeping with a pre-modern or modern viewpoint of the subject? What is your opinion? Support your stance with examples from the readings and the literature. How might a course that does not rigorously treat limits retain a modern perspective?

Exercise 1.4. Give an example of a function that is defined for all real numbers but continuous nowhere.

Exercise 1.5. Give² an example of two functions f and g so that

- $\lim_{y\to b} f(y) = L$,
- $\lim_{x \to a} g(x) = b$, and
- $f \circ g(x)$ is continuous,

but $\lim_{x \to a} f \circ g(x) \neq L$.

 $^{^1\}mathrm{In}$ his preface to the 2007 edition of Otto Toeplitz's Calculus: A genetic approach.

²From *Counterexamples in Analysis*, by Gelbaum and Olmstead.