MAT 53353: ADVANCED PERSPECTIVES ON THE CALCULUS HOMEWORK 2

Exercise 2.1. Construct a sequence with three limit points.

Exercise 2.2. Let A denote the set of all algebraic numbers. Demonstrate that this set is not (sequentially) complete.

Exercise 2.3. Let A denote the set of all algebraic numbers. Show that this set is countable.

Exercise 2.4. Demonstrate that inverse images of functions preserve set operations, e.g.

$$f^{-1}(A \cap B) = (f^{-1}(A)) \cap (f^{-1}(B)),$$

but that forward images do not.

Exercise 2.5. Weierstrauss argued that a real number could be regarded as a (possibly infinite) sum of an integer and unit fractions (i.e. positive fractions with a 1 in the numerator). Was he correct? Why or why not? Note that his argument postdates Cauchy's but predates Cantor's and Heine's refinement of the sequential argument.