## MAT 53353: ADVANCED PERSPECTIVES ON THE CALCULUS HOMEWORK 5

Exercise 5.1. Consider the elipse

$$\frac{x^2}{16} + y^2 = 1.$$

Use the method of exhaustion to approximate the area bounded by this curve to within one one-hundredth of the true value.

**Exercise 5.2.** A function F is a *primitive* of f if

$$\frac{\mathrm{d}}{\mathrm{d}x}F(x) = f(x).$$

Find a Riemann integrable function that has no primitive. Conclude that integration and differentiation are not inverse processes.

**Exercise 5.3.** Suppose that  $f : \mathbb{R} \to \mathbb{R}$  is continuous, positive, and decreasing. Show that

$$\lim_{b \to \infty} \int_0^b f(x) \, \mathrm{d}x$$

exists if and only if

$$\lim_{N\to\infty}\sum_{n=1}^N f(n)$$

exists.

**Exercise 5.4.** Give an example of a function that is zero outside the unit interval, has countably many discontinuities, and is *still* Riemann integrable.