Manual Construction of Continuous Cartograms through Mesh Transformation[1]

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Abstract

A computer-assisted framework is proposed to support the manual construction of cartograms. The framework employs a joint triangulation, similar to that used in rubber-sheeting, to define a piecewise affine transformation between map space and cartogram space. This guarantees preservation of all topological relations within and among transformed datasets with insertion of a finite number of points. To support intuitive user control of cartogram appearance, methods are developed to translate generically defined user adjustments of the cartogram into mesh vertex positions on either the source map mesh or cartogram mesh. The framework is implemented in a working prototype application and used to create sample cartograms of the United States and China. Results are compared with cartograms produced using diffusion and carto3f algorithms in terms of accuracy, aesthetic appearance and approximate construction time. Qualitative aspects of the manual construction process are also discussed.

Keywords

1.   Introduction

Cartograms have piqued the interest of geographers for over a century due to their unique form. Raisz (1936, p. 295) argued that they "set right common misconceptions held by even well informed people." When used to portray population, they show a human cartography (Dorling and Ballas 2011) and provide an alternate base map that allows spatial inferences to be made based on event rates per individual rather than per area (Sui and Holt 2008).

Constructing effective cartograms, however, is a hard problem both computationally and aesthetically. On the computational side, much research has been conducted over the past 45 years to develop automated procedures (algorithms) for cartogram construction (e.g. Dougenik et al. 1985, Tobler 1986, Dorling 1993, Gastner and Newman 2004, Henriques et al. 2009, Inoue et al 2009, Sun 2013, Tang 2013). However, understanding the cognitive properties of cartograms and improving their appearance such that they produce less cognitive dissonance remains a primary challenge in cartogram research (Sui and Holt 2008).

Broadly speaking, cartograms can be constructed by automated algorithms, semi-automated algorithms with parametric control, or manual (computer-assisted or hand-drawn) techniques. The vast majority of research to date has focused on the development of automated algorithms with or without adjustable parameters. In contrast, very little research has been conducted on the development of techniques to facilitate manual cartogram construction. There are several reasons, however, to believe that such research might be fruitful. Many cartograms developed before the computer era (Raisz 1934, Brinton 1939) are still appreciated for their aesthetic qualities (Krygier 2010). While cognitive studies are limited, preliminary studies indicate user preference for manually constructed block cartograms (Sun and Li 2010) and rectangular cartograms (Reyes and Juhász 2015) which are often produced by hand. Additional evidence comes from the continuing appearance of block cartograms (e.g. Straumann 2013, TeaDranks 2015) and other manually constructed cartograms (e.g. Taylor 2001, Archer et al. 2014) despite the widespread availability of user friendly algorithmic software that can produce cartograms at the push of a button.

The lack of research into techniques to facilitate manual construction suggests that many potential solutions to the aesthetic challenges of cartogram design might exist that remain

undiscovered. Creative cartographers interested in developing cartograms by hand have not had the benefit of computer-assisted design environments which could dramatically reduce the time and effort required to produce a cartogram. Although so-called "block" cartograms can be produced by resourceful cartographers using common graphics software, they are coarse, blocky and do not define a continuous transformation.

The objective of this paper is to propose and assess a computer-assisted design framework to facilitate manual construction of continuous cartograms. The proposed framework employes a joint triangulation (Saalfeld 1987, 1988) to define a piecewise linear transformation of map space into cartogram space. It will be demonstrated that this simple framework is sufficient to support accurate and detailed cartogram construction, without the use of quadtree-type structures. Furthermore, topological properties of any vector dataset can be preserved exactly through simple insertion of a finite number of vertices. This is notable because maintaining the topology of discrete representations of features has been a non-trivial problem in the development of automated cartogram construction algorithms and software (Tobler 2005, Andrieu et al. 2008, Sun 2013b).

The paper proceeds as follows. Section two reviews existing algorithms and manual cartogram construction techniques. The third section formally describes the mesh transformation framework. A working prototype software application is described in section four. In the fifth section, examples of manually constructed cartograms of the USA and China are presented and compared with cartograms produced using the diffusion and carto3f algorithms in terms of accuracy, aesthetics and construction time. The paper concludes by discussing the qualitative process of manual cartogram construction, potential advantages and limitations, and directions for further research.

2.   Cartogram Construction Techniques

There are many types of cartograms, but all share the characteristic that values associated with features of interest are represented by sizes of the representations of those features on the map (Olson 1976). Most commonly the attribute values of polygonal enumeration units are represented by their area, and thus a cartogram is often referred to as a "value-by-area" map. By convention, the cartogram variable is referred to as the *population*, which may be meant literally (i.e. the number of people in a district) or figuratively as any quantity or mass variable, including variables such as health spending and total rainfall (Dorling 2007).

Value-by-area cartograms come in many forms, including rectangular (Raisz 1934), non-contiguous (Olson 1976) and circle (Dorling) cartograms (Dorling 1993). Most research, however, has focused on construction of *contiguous* cartograms that preserve adjacency relations (Sui and Holt 2008). A cartogram is further said to be *continuous* if it defines a one-to-one correspondence between map and earth locations, thus functioning as a type of projection (Tobler 1963, 2005). In contrast, in non-continuous cartograms the boundaries of enumeration units are transformed, but the transformation of interior points remains undefined. For brevity, the term "cartogram" will be used here to refer to contiguous (but not necessarily continuous) value-by-area cartograms.

2.1. Automated Construction

Computer algorithms for constructing cartograms have been traced back to the 1960s, when cartogram software was presented to the computer graphics group at Harvard University (Tobler 2005). Early algorithms applied force to arbitrary grid cells but converged slowly. Speed improved dramatically with Dougenik et al.'s (1985) rubber-sheet algorithm that applied forces directly to polygon vertices. Public availability of software tools to implement the rubber-sheet algorithm (Jackel, 1997, Du and Liu 1999) and later the diffusion algorithm of Gastner and Newman (2004, implemented in Andrieu et al. 2008) led to increased accessibility and popularization of the map form (e.g. Barford and Dorling 2006).

Much research has been conducted to improve the speed, accuracy and shape-preserving qualities of cartogram algorithms. Diffusion cartograms in particular, which are built on a physics analogy between the "flow" of population and the diffusion of gases toward a state of uniform

density, have been said to provide the best balance of computational and aesthetic qualities (Sui and Holt 2008), although the claim that they minimize distortion in an absolute sense (Dorling and Ballas 2011) has not been supported. Recently, quadtree data structures have been used to further improve the speed of the rubber-sheet algorithm (Sun 2013a,b), and algorithmic construction of cartograms is likely to become even easier and faster in the future.

It is important to note that not just one but an infinite number of possible cartograms can be produced for any given population dataset (Tobler 2005). While increases in algorithm speed have made cartogram construction more feasible, improving shape recognition and aesthetic qualities remains a primary issue in cartogram research (Sui and Holt 2008). However, a number of studies have explicitly sought algorithms to improve the appearance or shape-preserving properties of cartograms. Several of these preserve rectilinearity, replicating popular forms in early hand-drawn cartograms. For example, pseudo-cartograms (Tobler 1986) are created by applying continuous horizontal or vertical deformation, presenting parallels and meridians as straight lines if the base map is in a cylindrical projection. Algorithms have also been developed to construct rectangular (van Kreveld and Speckmann 2007, Inoue et al. 2009) and rectilinear (Inoue et al. 2009) cartograms. Non-rectilinear strategies for preserving shape include limiting angular distortion (Inoue and Shimizu 2006) and utilizing scan lines (Keim et al. 2004) and medial axes (Keim et al. 2005) to control the direction of deformation. These methods produce cartograms with some appealing properties that are visually distinct from rubber-sheet and diffusion cartograms, but their aesthetic qualities have not been tested.

2.2. Manual Construction

Although cartogram algorithms have been reviewed by many authors (e.g. Tobler 2005), manually constructed cartograms have received less attention. This is unfortunate, since many hand-drawn cartograms contain unique styles that are not replicated by current algorithms. Ten examples of cartograms of the U.S.A. produced between 1911 and 1938 are documented by Krygier (2010), six of which were taken from Brinton (1939). Tobler (1963) provides two additional examples of U.S. cartograms and one world cartogram. Several hand-drawn cartograms of South American countries can also be found in Wilkie (1974). A visual assessment of these reveals several interesting features. First, geodesic and loxodromic boundaries are nearly always represented as straight lines, or occasionally as smooth, gradual curves. Second, many of these cartograms show additional landmarks to aid in interpretation, such as the Great Lakes and adjacent coastlines of Mexico and Canada. Raisz's (1934) rectangular cartogram similarly includes the Mississippi River and Appalachian and Rocky mountain ranges as reference landmarks. Third, in addition to state-level population data, several cartograms also portray major cities and/or emphasize sub-state features such as Cape Cod and Manhattan. Thus, these early cartograms seem to reflect a hierarchical cognitive model of space in which straight-line boundaries and perceptually salient landmarks play an important role.

Cartographers have continued to construct cartograms manually despite the advent of computer algorithms. The most common technique is the block method, in which population polygons are discretized to regular (e.g. square) grid cells which are then re-assigned to polygons to achieve the desired areas. Recent examples include world cartograms of population (TeaDranks 2015) and AIDS (Mendel 2005) and a map of carbon dioxide emissions on a world population cartogram (Kowalsky 2014). To improve appearance, vector polygons may be redrawn over the resulting grid (Campbell 2001). The block method has also been used as a post-processing method to improve the appearance of cartograms produced algorithmically (Straumann 2013). Overall, the block method is simple and intuitive in concept, but can be frustrating and time-consuming to implement (Campbell 2001).

Some researchers have employed more creative methods of manual construction. In 1971, L. Skoda and J.C. Robertson created a detailed population cartogram of the 266 Canadian census divisions (Jackson 1974) using ball bearings and flexible metal dividers. Notably, the use of computers was considered but rejected – not for lack of computational power but due to the

perceived difficulty of developing a "satisfactory" algorithm (Corbett 2003, p. 2). Another notable example is Taylor's (2001) cartogram of world "city space," which was manually constructed using the graphics tools in a word processing program (P. Taylor, pers. comm.). Uniquely among hand-drawn cartograms, it is characterized by broad elliptical curves, designed to emphasize a large hole in the traditional "heartland" region of central Asia. More recently, a manually constructed cartogram of the U.S. appeared on the cover of the 2012 Atlas of Elections (Archer et al. 2014). Dissatisfied with the results of computer algorithms, the cartogram was derived from a standard Alber's equal-area projection by manually repositioning boundary points and recalculating cartogram error (J. Clark Archer, pers. comm.).

An important limitation shared by all current methods of manual cartogram construction is that they do not define a continuous transformation. Thus, they cannot be considered map projections as suggested by Tobler (2005). This stands in contrast to algorithmic methods, and is significant because it becomes difficult if not impossible to add additional geographic details after a cartogram has been constructed. An ideal framework for cartogram construction should define a continuous transformation that can be recorded and subsequently applied to any geographic dataset, allowing features such as adjacent coastlines, rivers, and other reference landmarks to be added *post hoc*.

2.3. Cognitive Studies

Although cognitive studies are limited, there is some evidence that manually constructed cartograms may be preferred by map readers. In one study, manually constructed block cartograms were rated as more effective than three other types of cartograms by a pool of internet subjects (Sun and Li 2010). In terms of preference, the same study found that diffusion cartograms were rated slightly higher than block cartograms on average, but more people chose the block cartograms as their top choice than any other. In another study by Reyes and Juhász (2015), secondary school students given a choice between using a thematic map or cartogram to answer six analytical questions chose to use rectangular cartograms more often than diffusion cartograms; post-test comments were also largely positive for the rectangular cartogram but largely negative for diffusion cartograms.

With the exception of the above two studies, experiments have focused primarily on comparing cartograms with traditional maps (e.g. Kaspar et al. 2011, Sui and Holt 2008) rather than comparing different types of cartograms with one another. One reason for this may be the difficulty of producing multiple cartograms with distinct styles to serve as the basis for experimentation. It is notable, therefore, that both Sun and Li (2010) and Reyes and Juhász (2015) found significant differences among the different types of cartograms they tested. Improvement of manual cartogram construction techniques would support cognitive research by facilitating prototyping of a greater variety of cartogram designs.

2.4. Cartogram Error

Most cartograms contain error, and this must be considered when comparing alternative cartogram designs. Numerous cartogram error metrics have been proposed based on the relative sizes and populations of districts. Let $\{p_1, p_2, \ldots, p_n\}$ and $\{a_1, a_2, \ldots, a_n\}$ denote the population and area of each district relative to the overall total, such that $\sum p = \sum a = 1$. In a perfect cartogram, the proportions would be the same for each district, i.e. $p_i = a_i \ \forall i$. Usually, however, the correspondence will not be exact. To capture the difference, Inoue and Shimizu (2006) propose using the root mean squared error (RMSE), which may be written as:

$$\varepsilon_{RMS} = \sqrt{\sum (p_i - a_i)^2 / n} \tag{1}$$

RMSE is symmetric in the sense that a district will contribute the same amount to overall error if it is *x* times too large as if it is *x* times too small. It is not district-equitable, however, because two districts

that are both *x* times too large will contribute differently to overall error if their populations differ. Alam et al. (2015) present an error metric that is both symmetric and district-equitable:

$$\varepsilon_{SDE} = \sum \frac{|a_i - p_i|}{min(p_i, a_i)} / n \qquad (2)$$

A similar metric was employed as early as Dougenik et al. (1985). Not all authors agree, however, that a cartogram error metric should be district-equitable. Keim et al. (2005) argue that regions with higher population should have greater influence on the overall metric. However, this logic has not been formalized.

3.   Manual Construction Technique

Mathematically, a continuous cartogram is a bijective mapping function $M: \mathbb{R}^2 \to \mathbb{R}^2$ that translates any point from map space into cartogram space. Since the composition of two bijective functions is also bijective, a generic approach to cartogram construction might be to decompose it into smaller transformations $\{M_1, M_2, M_3, etc.\}$. A user interface could provide tools for defining local transformations (e.g. "move this boundary line up a little to the left and rotate it"), and record these as a series of functional mappings that in combination define the cartogram. Practically speaking, a fully generic approach would invite problems of interoperability as different software programs would use different functions to define local transformations. Furthermore, such a generic approach would invite topological errors caused by discrete approximations of continuous functions, and/or unchecked increases in data size as extra vertices are inserted to avoid topological errors.

3.1. Triangular Mesh Transformation

To avoid problems of interoperability and discretization of continuous functions, a more structured but still highly flexible approach is to define a piecewise linear (affine) transformation using a pair of triangular meshes (i.e. triangulated irregular networks or TINs). Such joint triangulations are widely used for rubber-sheeting to align map layers (Gillman 1985, Saalfeld 1988), and have been discussed in the context of automated algorithms (Inoue et al. 2007). Similar hexagonal meshes are also illustrated by Tobler (1973, 2005). However, such a framework has not been employed in support of manual cartogram construction.

A joint triangulation is used here to support manual cartogram construction by two-step process in which (1) the cartographer performs adjustments to the cartogram, and (2) these adjustments are simulated by altering the vertex positions of the underlying meshes. Thus, the triangular meshes provide a finite, well-defined data structure to record the changes applied by the cartographer. Formally, the cartogram transformation is defined as a bijective mapping function $M_{mc}: \mathbb{R}^2 \to \mathbb{R}^2$ prescribed by two sets of non-overlapping triangles, one (T_m) on the source map and the other (T_c) on the cartogram. T_m and T_c are constrained as follows:

1. T_m and T_c must be topologically equivalent
2. Vertices of all triangles in both T_m and T_c must be wound clockwise
3. Coordinates of all exterior vertices in T_c must equal the corresponding vertices in T_m

This formulation represents a minimal set of constraints, but leaves open specifics such as vertex degree, geometric constraints and the manner in which mesh vertices are adjusted.

Given a transformation mesh conforming to the above constraints, the bijective function M_{mc} is defined by the following algorithm, which transforms a standard vector geographic dataset consisting of sequential vertices V_m in the map projection to a corresponding set of sequential vertices V_c in the cartogram space:

1. $V_c \leftarrow$ copy V_m
2. $V_c \leftarrow$ insert vertices into V_c at each intersection of V_c and T_m
3. For each vertex v in V_c:
    3.1. $t \leftarrow$ triangle in T_m containing v
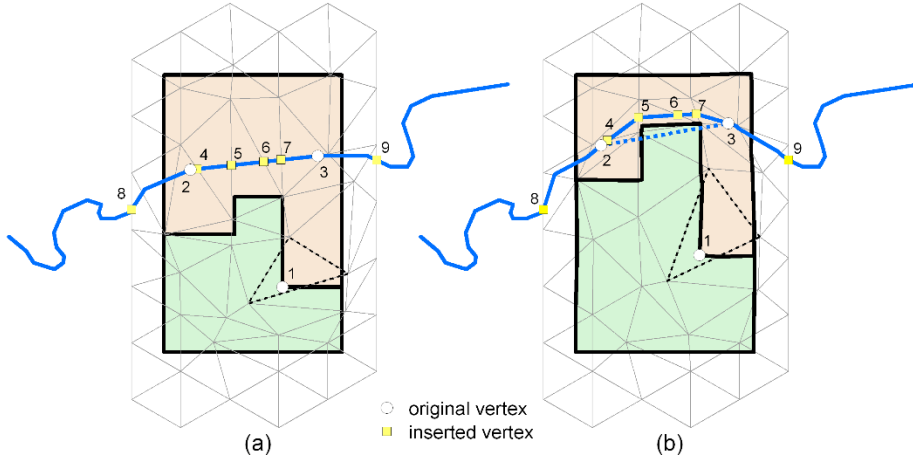   3.2. if t is not null:

Figure 1: Features of a cartogram transformation mesh. (a) map triangles T_m and original data, including two districts and a river; (b) cartogram triangles T_c and transformed data. See text for explanation of numbered points.

     3.2.1. $v_b \leftarrow$ barycentric coordinates of vertex v in t
     3.2.2. $t' \leftarrow$ triangle in T_c corresponding to t
     3.2.3. $v \leftarrow$ Cartesian coordinates of v_b in t'

An example scenario is illustrated in Figure 1. The scenario consists of two real-world districts (Fig. 1a), with the lower (green) district having a higher population density. A pair of transformation meshes (gray triangles) is constructed to enlarge the lower district, resulting in the cartogram in Fig. 1b. In this example, each interior node has degree 6, and the geometry of both meshes is irregular.

The process of coordinate transformation can be understood by considering the district boundary vertex (1) located in the highlighted mesh triangle (dashed line) in Fig. 1a. The perpendicular distances from this vertex to the triangle edges define unique barycentric coordinates. When transferred to the corresponding triangle in the cartogram mesh (Fig. 1b), these barycentric coordinates define the location of vertex (1) on the cartogram.

Maintenance of topological relations between all transformed features is guaranteed because (a) T_m and T_c are constrained to be topologically equivalent, (b) topological preservation within triangles is guaranteed by the properties of an affine transformation, and (c) topological preservation along the edges between triangles is guaranteed by the insertion of vertices at intersections with mesh edges in step 2 of the transformation algorithm. An example of a topological error that would occur without the insertion of these vertices is illustrated by the stream in Fig. 1. The discrete representation of the stream on the source map (Fig. 1a) contains a long segment extending between vertices (2) and (3). A straight line segment between the corresponding locations on the cartogram (dashed blue line in Fig. 1b) crosses the district boundary, creating a potential topological error. This is avoided by inserting vertices at the intersections between the stream segment and edges of T_m (points 4-7). The result is that the transformed stream on the cartogram stays within the upper district, preserving the correct topological relations.

The framework also ensures continuity between the internal and external space of the transformation. This is illustrated by the river in Fig. 1, which extends beyond the space enclosed by the triangles in T_m. The conditional statement in step 3.2 of the transformation procedure ensures that the coordinates of feature vertices external to T_m are unaffected by the transformation. Constraint 3 further ensures that locations on the outer edges of T_m (e.g. points 8 & 9 in Fig. 1) have the same coordinates on the source map and cartogram. As a result, continuity is maintained for

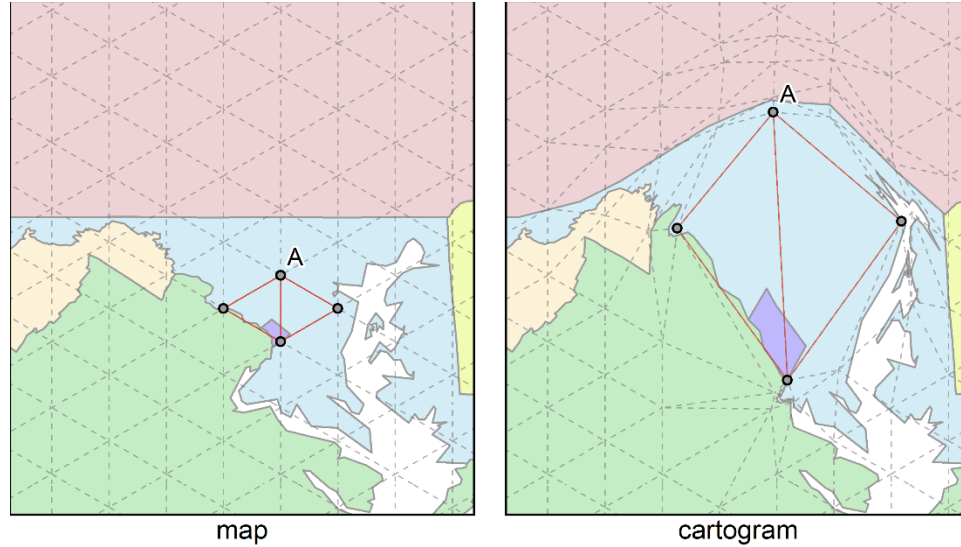|	
map	cartogram

Figure 2:   Enlargement of Washington DC by adjustment of cartogram triangle vertices.

features that extend beyond the extent of the cartogram meshes, ensuring that any feature can be brought into the cartogram *post hoc*.

3.2. Manual adjustment of mesh vertices

Manual cartogram construction within the above framework requires an interface to interactively manipulating mesh vertices while simultaneously monitoring shape distortion effects and cartogram error. Two general approaches are possible. Tools can be provided to select and reposition mesh vertices directly, in which case the cartographer must anticipate the effect on the resulting cartogram. Alternatively, tools can be provided to "reshape" the cartogram, in which case mesh vertex locations must be computed to most closely achieve the desired changes. In either case, a choice must be made as to whether to reposition vertices in T_m or T_c.

If vertices are manipulated directly, repositioning vertices in T_c is more intuitive because the direction of movement of a given location on the cartogram will be identical to the direction of movement of mesh vertices. However, the disadvantage of this approach is that it is difficult to enlarge small, densely populated districts without adversely affecting the size and shape of neighboring districts. For example, Fig. 2 shows a possible repositioning of cartogram vertices to enlarge Washington D.C. (the small district near the center of the map). Although the district occupies only a portion of each of the highlighted triangles, it cannot be enlarged without enlarging the entire triangles. As a result, vertex A is pulled unnecessarily far northward, inadvertently altering the shape of the Pennsylvania-Maryland boundary. Because the map area of each triangle is fixed, this problem is not easily avoided. The only solution is to densify the mesh either in its entirety or locally using a quad-tree structure. In either case, mesh storage size and computation time will increase, resulting in a less responsive user interface.

An alternative is to reposition vertices in T_m (Fig. 3). In this case, the direction of movement of cartogram locations will be opposite that of mesh vertices, resulting in a somewhat unintuitive user interface. However, this approach provides greater flexibility for handling small, densely populated districts. In Fig. 3a, triangle vertices are moved into densely populated Washington D.C., with the effect that district boundaries on the cartogram are pulled outward. One way to understand this is to observe that each small red triangle on the map in Fig. 3a is transformed to a corresponding large red triangle on the cartogram in Fig. 3b. The map triangles are arranged to cover
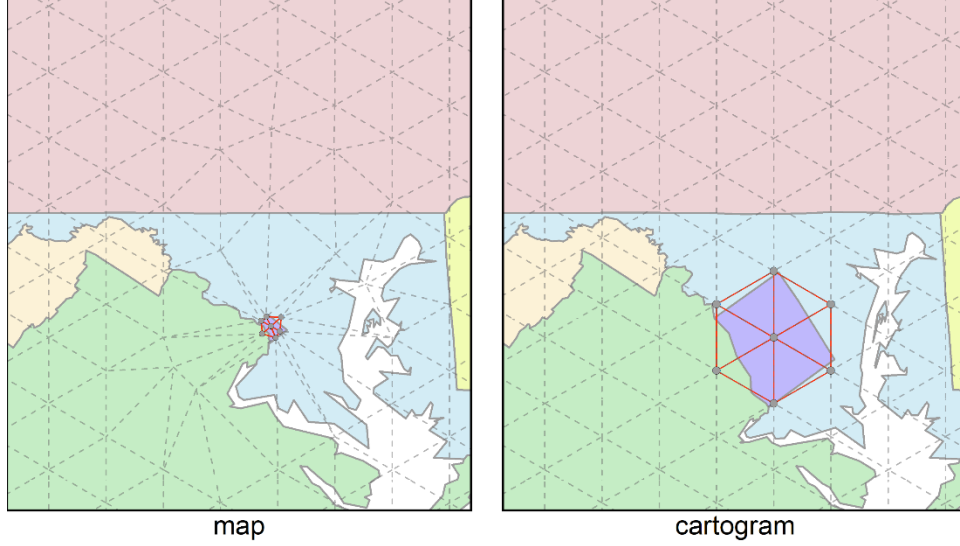
Figure 3: Enlargement of Washington DC by adjustment of map triangle vertices.

very little area outside of the district, minimizing the effect on neighboring states and allowing the linearity of the Mason-Dixon line to be preserved.

Repositioning mesh vertices on the map in order to effect the reverse movement of district boundaries on the cartogram is unintuitive at first, but a useful analogy to a dot-density map can facilitate conceptual understanding. If the cartogram mesh triangles are fixed in a regular hexagonal pattern as in Fig. 3, then each triangle will contain the same area on the cartogram. Since a cartogram defines a space of uniform population density, it follows that each triangle on an accurately constructed cartogram will represent a constant population. Knowing this, the cartographer must seek to cover each district on the map with the correct number of triangles. In Fig. 3, the mesh has been designed such that each triangle represents approx. 100,000 people, and therefore Washington D.C. (population: 638,000) should be covered with a bit more than six triangles. A similar calculation can be made with mesh vertices.

3.3. Automated adjustment of mesh vertices

Manual adjustment of mesh vertices may be tedious even if the process is grasped conceptually, and is one step removed from the cartographer's vision of altering the cartogram's appearance. An alternative approach is to provide tools to allow the cartographer to directly "mold" the cartogram like a piece of clay, specifying desired changes to the cartogram through any number of local adjustment tools. To conform to the proposed mesh transformation framework, user-specified changes to an existing (prior) cartogram using such a tool must be translated into new mesh vertex coordinates defining a new (posterior) cartogram.

Generally, let $M_{cc'}: \mathbb{R}^2 \to \mathbb{R}^2$ represent a user-specified adjustment to the cartogram, defined as a bijective mapping from a prior cartogram space to a posterior cartogram space. Given prior triangle vertices U_m and U_c on the map and cartogram, respectively, the problem is to compute posterior triangle vertices $U_{m'}$ and $U_{c'}$ such that the mesh transformation that they define ($M_{m'c'}$) is approximately equivalent to the combination of the user-specified adjustment ($M_{cc'}$) and the prior mesh transformation (M_{mc}), i.e. $M_{m'c'} \approx M_{cc'} M_{mc}$. The problem space is infinite, so we search for an approximate solution by taking a finite sample. Let us define:

X_c a finite sample of locations on the prior cartogram

X_m corresponding locations on the map $\left(= M_{cm}(X_c)\right)$

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$X_{c'}$ corresponding target locations on the posterior cartogram $\left(= M_{cc'}(X_c)\right)$

If only vertices on the cartogram mesh are adjusted, the following optimization problem is specified:

Given: $U_m, X_m, X_{c'}$
Find: U_c'
Minimize: $\|M_{mc'}(X_m) - X_{c'}\|$

To simplify, the prior mesh vertices can be chosen as sample locations ($X_c = U_c$). It follows that the corresponding points on the map are mesh vertices (i.e. $X_m = U_m$), and since $M_{mc'}(U_m) = U_{c'}$, the solution is trivial:

$$U_{c'} = X_{c'} \tag{3}$$

The problem is slightly more complicated if we choose to reposition vertices on the map. In that case, the optimization problem can be stated as:

Given: $U_c, X_m, X_{c'}$
Find: U_m'
Minimize: $\|M_{m'c}(X_m) - X_{c'}\|$

To simplify the solution, we choose sample points X_c that would be transformed into the locations of cartogram vertices under the user-specified adjustment, i.e. $M_{cc'}(X_c) = U_c$. The inverse of $M_{cc'}$ must exist since it is a bijective function; let us denote this inverse function as $M_{c'c}$. Then the selected sample locations are computed as:

$$X_c = M_{c'c}(U_c) \tag{4}$$

The corresponding points on the map are:

$$X_m = M_{cm}(X_c) \tag{5}$$

Since the points in X_m correspond to the cartogram locations that the user has specified should be moved to U_c, and since the posterior mapping $M_{m'c}$ will transform $U_{m'}$ to U_c, the optimal solution is to move the map vertices to these locations, i.e. set:

$$U_{m'} = X_m \tag{6}$$

Whether choosing to move vertices on the map or cartogram, care must be taken to avoid topological errors in the posterior triangular meshes. When mesh vertices begin in a geometrically regular arrangement (e.g. a hexagonal lattice), the solutions to eq. 3 and eq. 6 will usually produce a topologically correct posterior mesh if the user-defined adjustment is broadly smooth and continuous. However, topological errors in the posterior meshes become more likely if either prior mesh contains long, skinny triangles with angles near 180°, as may be caused by previous user adjustments. The problem of maintaining topological relations in a joint triangulation has been noted by previous researchers (Gillman 1985, Saalfeld 1987). It would not occur in a second-order continuous transformation but is the result of discretization, and the likelihood of occurrence decreases with mesh density. Saalfeld (1987) illustrated a method that can be used to guarantee topological consistency in a joint triangulation by inserting a finite number of new vertices, but the number of added vertices is potentially large. An alternate strategy is to disallow any user adjustment that results in a topological error in mesh vertices, as indicated by the presence of a triangle with vertices wound counterclockwise.

4.   Prototype user interface implementation

Based on the above framework, a prototype software application was developed to support manual cartogram construction. The application was written by the author in Visual Basic.Net (Microsoft Corp.) with mapping functionality supported by the DotSpatial open source GIS library. A screenshot of the application is shown in Fig. 4.
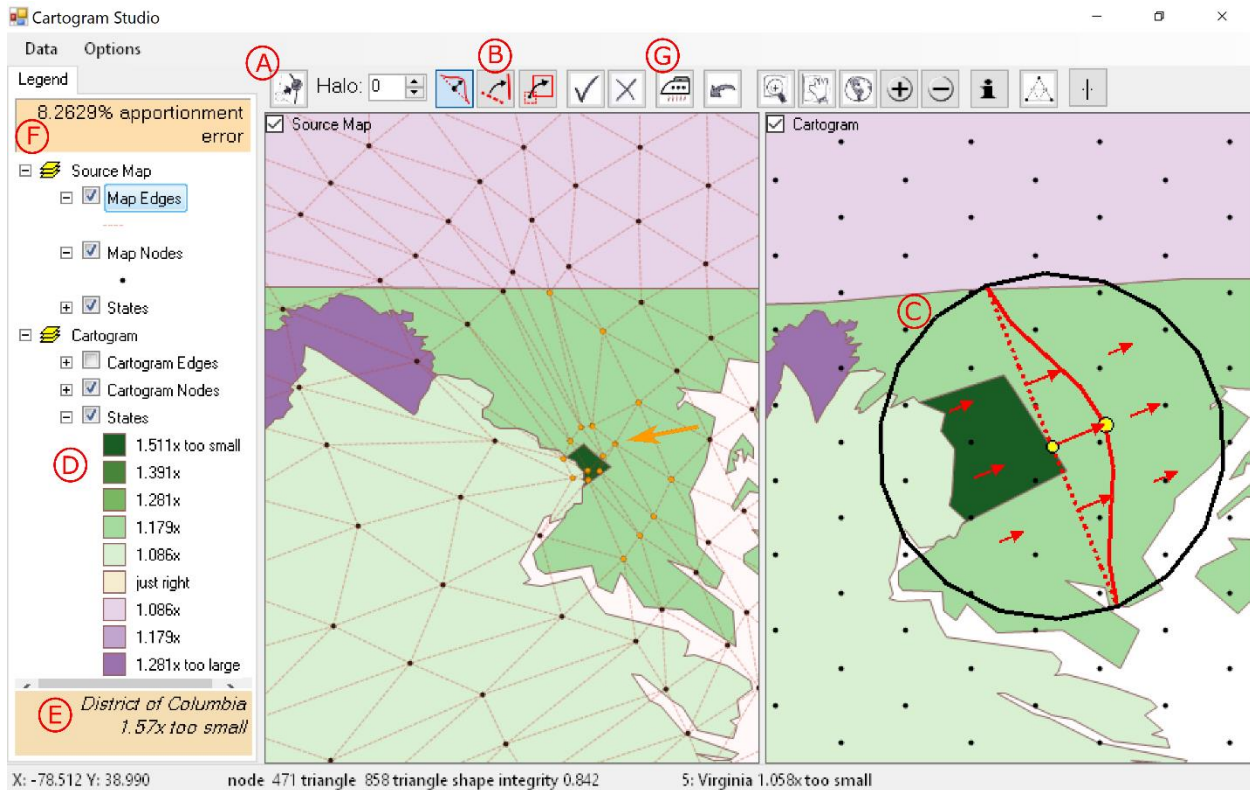
Figure 4: Application built to support manual interactive cartogram construction using the triangular mesh transformation framework. See text for description of lettered features.

The interface shows two linked map panels containing the source map (left) and cartogram (right). When a new dataset is loaded, an initial cartogram transformation consisting of a pair of identical triangular meshes is constructed, and the "cartogram" is nothing more than a copy of the source map. The actual cartogram is constructed by incrementally adjusting the transformation mesh vertices directly or indirectly using the interactive tools located in the toolbar above the maps. Fig. 4 shows a cartogram in progress in which only map mesh vertices have been altered and cartogram mesh vertices remain at their original, geometrically regular coordinate locations.

Direct repositioning of mesh vertices is supported with a vertex repositioning tool (Fig. 4a) which can be applied to vertices on either the source map or cartogram. For automated mesh vertex adjustment, three tools were developed to define local cartogram adjustments (Fig. 4b). These tools support targeted adjustment based on point, line and rectangle geometries respectively. To use these tools, the user first constructs a geometry on the (prior) cartogram and then repositions or alters the geometry to reflect the target shape on the posterior cartogram. An example with the point adjustment or "nudge" tool is illustrated (Fig. 4c). The target adjustment is defined by pressing, dragging and releasing the mouse; the yellow points represent the press and release points. The adjustment must be continuous, so a local bijective function is defined that allows target destinations for any point in a local neighborhood to be computed. The red arrows in Fig. 4 depict a sampling of the movement vectors defined by this function. Also, the curved red line in the figure represents the target destinations of the set of points comprising the dashed red line. The black circle represents the bounds of the local bijective function, beyond which the cartogram will be unaffected.

Fig. 5 shows the result of applying the point adjustment specified in Fig. 4, with the resulting enlargement of Washington D.C. In this example, the mesh vertices highlighted in orange were
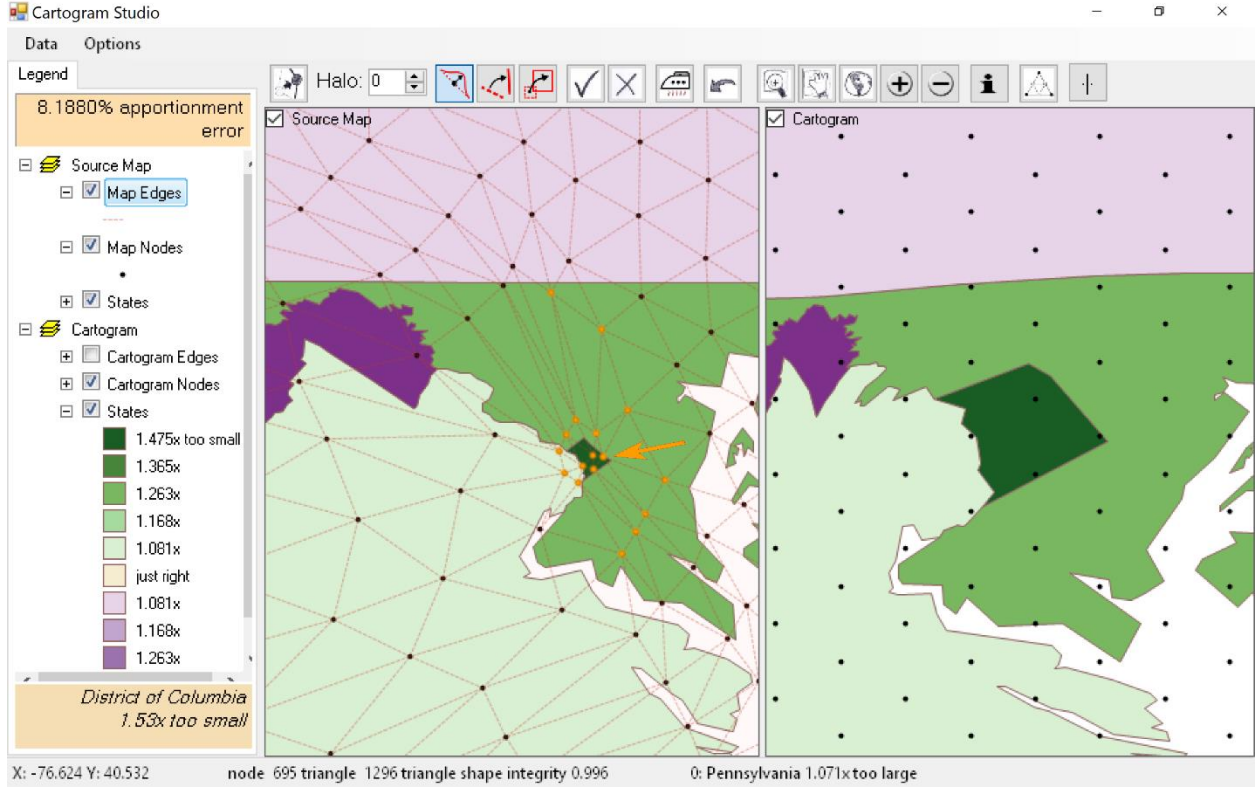
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Figure 5:   Application interface showing the cartogram after applying the "nudge" transformation shown in Fig. 4.

repositioned on the map using eqs. 5-7. An example of this repositioning can be seen with the vertex indicated by the orange arrow in the left-hand map on both figures. In Fig. 4, this vertex is located well to the north and east of Washington D.C., but after adjustment (Fig. 5) it is repositioned inside the district. Optionally, the user may choose to reposition vertices on the cartogram mesh instead of the source mesh (not shown).

Several methods are used to provide feedback on cartogram error. Population districts are automatically assigned to error classes portrayed using a color-blind friendly diverging purple-green color scheme (Brewer 2016). A legend indicates the average size of districts in each class relative to the correct size for the designated population variable (Fig. 4d). For a more exact value, users can hover or click on a district and view the size ratio for that district (Fig. 4e).

A metric to report overall cartogram error was developed by extending the argument that higher population districts should have greater influence on error (Keim et al. 2005). Since an accurate cartogram assigns equal area to each individual, a cartogram error metric should be weighted equally by individual rather than by district. This concept is illustrated in Fig. 6, which shows two alternative cartograms for a hypothetical set of three districts with total population 10. While the distribution of errors among districts differ, the number of persons affected by cartogram error is equal in a quantifiable sense. Specifically, if individuals (labelled circles) are assigned equal area and placed optimally in districts, exactly two people must be placed in the wrong district on both cartograms.

Building on this concept, an *apportionment error* is defined as follows:

$$\varepsilon_{APP} = \sum |p_i - a_i|/2 \qquad (7)$$
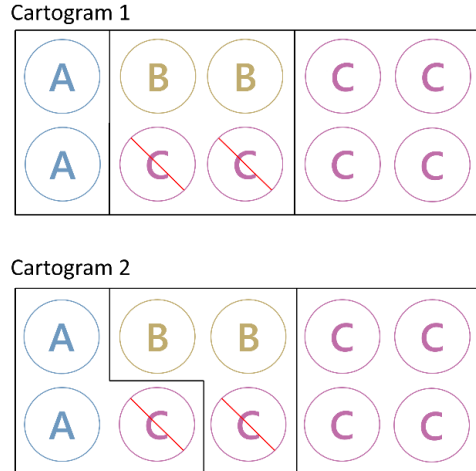
10

Figure 6:   Illustration of the calculation of *apportionment error* for two cartograms produced from three districts {A,B,C} with populations {2,2,6}. Relative district areas in the two cartograms are {0.2,0.4,0.4} and {0.3,0.3,0.4}. In both cartograms, two out of ten individuals must be placed in the wrong district, resulting in an apportionment error of 0.2.

The value indicates the proportion of the overall population that is "apportioned" to the wrong district by the cartogram. Apportionment error can range from 0 to 1. The denominator avoids double-counting since for each district that is one person "short" there is a corresponding district that has one "extra" person. In the example in Fig. 6, apportionment error is 0.2 for both cartograms.

Before committing any user-defined cartogram adjustment, the application checks for topological errors in the posterior meshes by looking for adjusted triangles with vertices wound counterclockwise. If an error is found, the user is informed that the adjustment cannot be completed and nodes associated with triangles with topological errors are highlighted. The user may manually reposition vertices to make these triangles more regular and then attempt the adjustment again. Alternatively, a tool is provided for "ironing out" these skinny triangles (Fig. 4g). This tool moves a designated vertex to the centroid of the kernel of its surrounding hexagon, which defines the region in which the vertex can be repositioned without causing topological errors (Lee and Preparata 1979). All symbols and values are updated automatically after each cartogram adjustment.

5.   Examples

Two cartograms are presented to illustrate the proposed manual construction framework and prototype application. Both examples were constructed manually by the author. Mesh adjustment was confined to the source map meshes, in part to facilitate handling of small, densely populated districts but also for simplicity and to preserve the dot-density metaphor. Since the proposed framework is a design environment and not an algorithm, the examples represent just one of infinitely many possibilities, and other cartographers might produce very different cartograms using the same data and software tools. Nevertheless, the examples are sufficiently complex to identify potential advantages and limitations of the proposed manual construction framework.

For comparison, cartograms were also produced using Scapetoad software (Andrieu et al. 2008), which implements the diffusion algorithm of Gastner and Newman (2004), and Software for Unified Network Analysis (SUNA; Sun 2016) which implements the carto3f algorithm (Sun 2013). For all cartograms, size errors of individual districts were measured as a/p (the ratio of proportional area to proportional population), and overall apportionment error was calculated using eq. 7. To provide an equal basis for comparison, an attempt was made to produce cartograms with roughly equal apportionment error. Shape characteristics of all cartograms were assessed qualitatively.
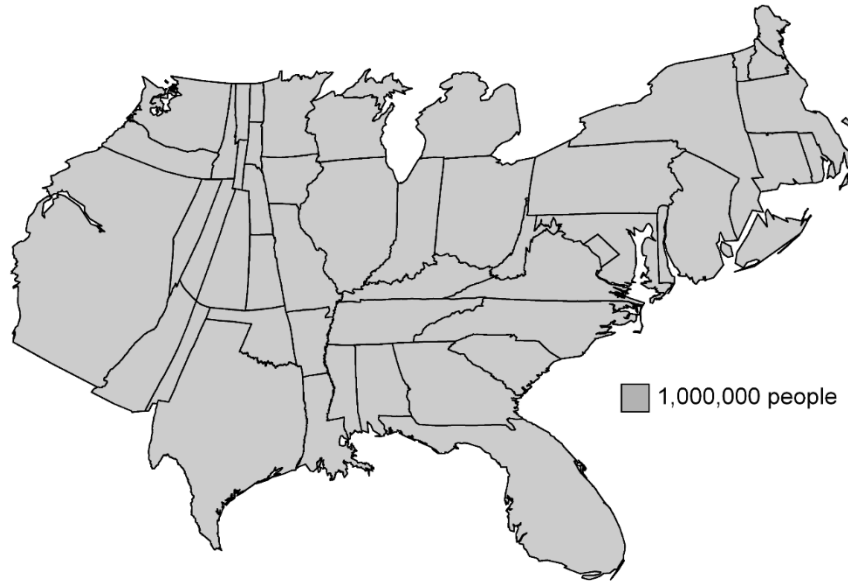
Figure 7:   Total 2010 population of the conterminous U.S. by states and the District of Columbia, areas proportional to population (cartogram). Produced by the author using prototype software for manual cartogram construction.

5.1. Population Cartogram of the USA

A manually constructed cartogram of the lower 48 conterminous U.S. states and the District of Columbia is shown in Fig. 7. The cartogram shows 2010 decennial census population data, and was constructed from a source map registered to a cylindrical equal area projection. Population densities in the projected coordinate system ranged from 2.3/km^2 for Wyoming to 3793.1/km^2 for Washington D.C. The cartogram was produced over a period of several months, in tandem with the development of the software design environment. The cartogram transformation is defined by a pair of meshes with 135,296 triangles and 68,193 vertices each, of which 11,323 vertices are inside the study region. The cartogram attempts to preserve linear boundaries and salient cognitive features, as well as the shapes of individual states. As with any manually constructed cartogram, Fig. 7 represents a work in process that may undergo continued refinement.

Fig. 8 shows cartograms of the U.S.A. produced in Scapetoad and SUNA from the same data. Default parameters resulted in relatively good aesthetic appearance with smooth borders (Fig. 8a, 8b). However, apportionment errors were substantially higher than the manually constructed cartogram, and so parameters were adjusted to reduce error. In Scapetoad, the cartogram grid size was increased from 200 to 3,000 and the diffusion grid size from 128 to the maximum allowed value of 1,024. In SUNA, the number of steps was increased from 4 to 5, quadtree depth was increased from 6 to 8, and the enlargement exaggeration rate was increased from 8 to 12. The resulting cartograms are shown in Figs. 8c and 8d.

5.2. Population Cartogram of Chinese Provinces

Population cartograms of Chinese provincial level administrative divisions ("provinces") constructed using both manual and automated techniques are shown in Fig. 9. All cartograms were constructed from 2010 census population data registered to a Lambert conformal projection. China contains 34 provincial divisions, including five autonomous regions, four province-level municipalities and two special administrative regions. As a result, population density varies more than among U.S. states, from 2.6/km^2 in Tibet to 50,209/km^2 in Aomen (Macau). Three Chinese provinces (Aomen, Hong Kong and Shanghai) have higher population densities than Washington D.C.
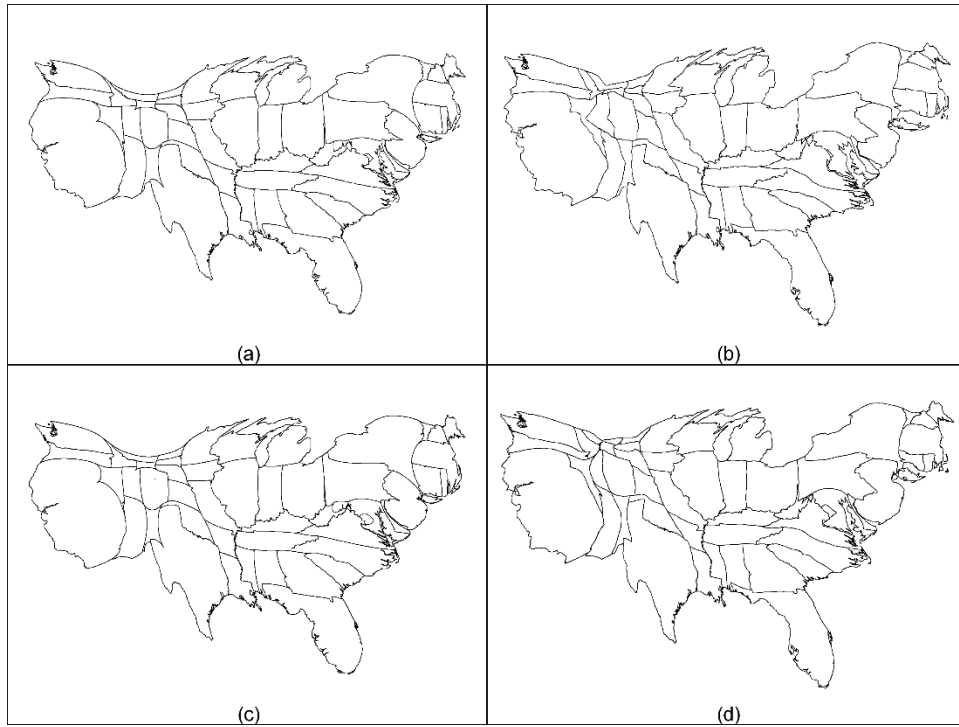
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Figure 8: Total 2010 population of the conterminous U.S. states and the District of Columbia, areas proportional to population (cartogram). Produced using (a) Scapetoad with default parameters, (b) SUNA using default parameters, (c) Scapetoad with adjusted parameters, (d) SUNA using adjusted parameters.

The manually constructed cartogram (Fig. 9a) was produced using meshes of 33,440 triangles and 16,981 vertices, of which 3,870 were internal to the study region. Since China contains no geodesic or loxodromic borders, the cartogram design was guided by broad curves, weight-balancing and shape preservation of specific salient features.
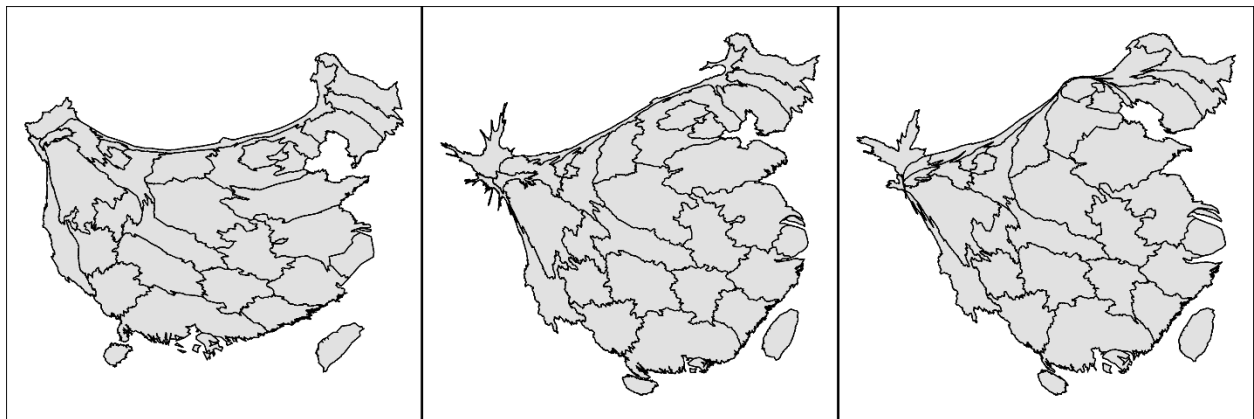

Figure 9: Total 2010 population of Chinese provinces and other first level administration units, areas proportional to population (cartogram). Constructed using (a) prototype software for manual cartogram construction, (b) Scapetoad, (c) SUNA.

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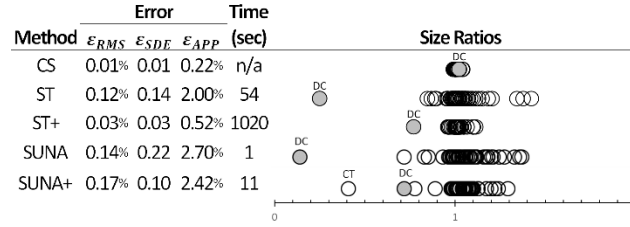| Method	Error ε_{RMS}	ε_{SDE}	ε_{APP}	Time (sec)	Size Ratios
CS	0.01%	0.01	0.22%	n/a	
ST	0.12%	0.14	2.00%	54	
ST+	0.03%	0.03	0.52%	1020	
SUNA	0.14%	0.22	2.70%	1	
SUNA+	0.17%	0.10	2.42%	11	

Figure 10: Overall apportionment error and size ratios of individual states for five cartograms created using: prototype software for manual cartogram construction (CS), Scapetoad with default parameters (ST), Scapetoad with adjusted parameters (ST+), SUNA using default parameters (SUNA), SUNA using adjusted parameters (SUNA+). Highlighted points represent Washington D.C.

6.   Assessment

To evaluate the relative merits of the manually constructed cartograms, accuracy, aesthetic qualities, and time and effort to construct each cartogram were assessed.

6.1. Accuracy

Fig. 10 shows overall error according to three metrics and the distribution of size ratios of each state for the U.S. cartograms. Errors were low on all cartograms, but the manually constructed cartogram had lowest error on all overall metrics. Parameter adjustment in Scapetoad reduced overall error to within 2-3x that of the manual cartogram, but parameter adjustment in SUNA led to only minor improvement.

Some insight can be gained by looking at the size ratios of individual states. In the manually constructed cartogram, these fell within a tight distribution around the ideal value. The small but densely populated Washington D.C. was 102% of its correct size, compared to 25% and 13% in Scapetoad and SUNA, respectively, using default parameters. Parameter adjustment in Scapetoad led to marked improvement but Washington D.C. remained an outlier at 77% of its correct size. SUNA uses an enlargement exaggeration factor that creates a tendency to overadjust. As a result, in some trial runs Washington D.C. ended up nearly equal to or even larger than the correct size, but other states were either too small or too large (data not shown). In the final cartogram selected for SUNA, the size ratio for Washington D.C. was 69%, but Connecticut was reduced to 39% of its correct size.

Overall error and size ratios of individual provinces on the cartograms of China are shown in Fig. 11. Values of ε_{RMS} and ε_{APP} were similar among the three cartograms, but values of ε_{SDE} were highest for Scapetoad and lowest for the manually constructed cartogram. This reflects the high penalty the ε_{SDE} metric placed on the incorrect proportions of the relatively small district of Aomen, which was just 2% and 4% of its correct size in the Scapetoad and SUNA cartograms respectively. In Scapetoad (but not SUNA) Hong Kong was also too small at 49% of its correct size. These small but densely populated provinces were both 99% of their correct size on the manually constructed cartogram.
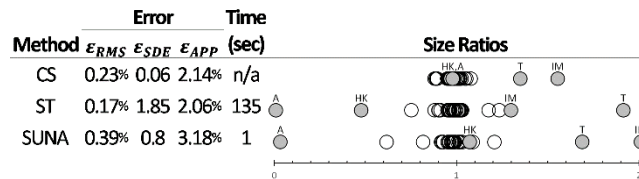
Method	Error ε_{RMS}	ε_{SDE}	ε_{APP}	Time (sec)	Size Ratios
CS	0.23%	0.06	2.14%	n/a	
ST	0.17%	1.85	2.06%	135	
SUNA	0.39%	0.8	3.18%	1	

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At the other end of the scale, the sparsely populated provinces of Tibet and Inner Mongolia were too large in all three cartograms. The highest size ratios were for Inner Mongolia in SUNA (202%) and Tibet in Scapetoad (193%). The size ratios of these provinces were also high in the manually constructed cartogram, and Inner Mongolia was larger in the manually constructed cartogram (157%) than in Scapetoad (131%).

6.2. Cognitive and Aesthetic Qualities

Previous researchers have judged the results of cartogram algorithms through visual assessment (e.g. Sun 2014a,b, Henriques et al. 2009, Inoue et al. 2009). Given the artistic nature of manual cartogram construction, any such assessment would naturally involve a high degree of subjectivity. Instead, a description of aesthetic choices and cognitive foci is given to provide insight into differences between manual and algorithmic construction as well as a basis for future cartographers to understand the cartograms presented here. It bears emphasizing that other cartographers might make very different design choices, and no general claim is made of aesthetic quality.

In constructing the cartogram of the U.S. (Fig. 7), many ideas were borrowed from historical hand-drawn cartograms listed by Krygier (2010). Most obviously, a great deal of attention was paid to the preservation of linear features, including individual boundary lines as well as salient visual sequences. For example, an attempt was made to visually extend the Virginia-North Carolina westward, tracing a rough line along the borders between several other pairs of states westward to the eastern edge of Nevada. Two related cognitive focii were the Oklahoma panhandle and the four corners states. The task of preserving these feature was especially difficult because the region is so sparsely populated. The panhandle was handled by greatly exaggerating the length of the Colorado-Oklahoma border. The upwards curve of the southern border of Utah in Fig. 7 represents a compromise between preserving the rectilinearity of the four corners, maintaining the "crook" in the California-Nevada boundary and achieving the correct areas for each state.

In addition to linear borders, attention was also paid to cognitive landmarks such as Cape Cod, the Northwest Angle and the "mitten" of Michigan's lower peninsula. For example, both Cape Cod and the "mitten" are preserved by maintaining and even exaggerating the water bodies separating them from nearby features.

Different from the USA, Chinese provinces lack geodesic or loxodromic borders that may serve as a visual reference. In addition, the author is not aware of any hand-drawn cartograms of China that could be consulted for reference. To provide guidance during the construction process, several design objectives were identified. The first was to balance the northeast and northwest corners of the country, which were thought to provide visual anchors for the bulk of the landmass. A second objective was to preserve the two broad curves that make up the northern border with Mongolia and the southeastern coastline from Shanghai to Hainan Island. The final objective was to preserve the shapes of the islands of Taiwan and Hainan and the Liaodong and Shandong peninsulas, considered to be important visual landmarks.

Some insight into how these design objectives were implemented can be gleaned from the map triangulation meshes (Fig. 12). Since the cartograms were based on state/province populations, mesh placement within each state or province as well as outside all districts was unconstrained and largely dictated by aesthetic considerations. Unusual concentrations of mesh vertices in some cases indicate areas of cognitive focus. This can be seen in the the dense concentrations of mesh vertices in the Oklahoma panhandle. An even denser concentration between Michigan's lower and upper peninsulas is visible from mesh edges (vertices outside states are not shown). Mesh vertices may also be intentionally spread out in a regular pattern as a means of
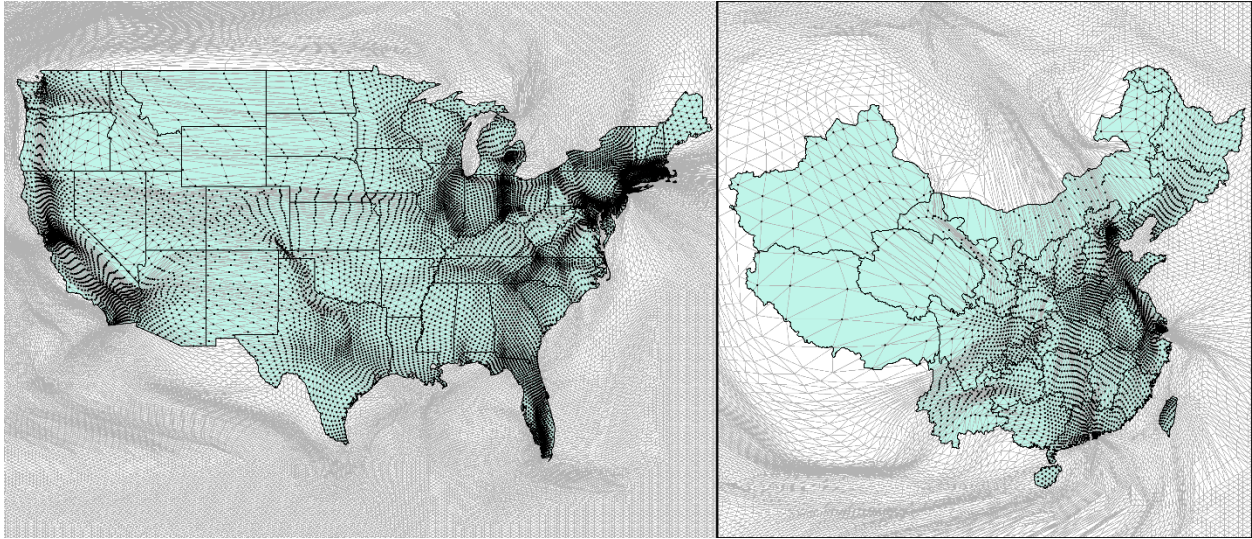
Figure 12: Source map triangle meshes used to construct cartograms of the USA and China. Since cartogram meshes were constrained to be geometrically regular, mesh vertices on source maps approximate dot-density maps.

preserving shape. This technique was employed in the southernmost Chinese island province of Hainan as a simple means of preserving the shape of the island.

6.3. Time and Effort

The effort required to construct a cartogram must be considered for pragmatic reasons when choosing among different techniques. Computation time for Scapetoad is measured automatically by the software. For SUNA, approximate computation times were recorded using a digital stopwatch. All computation was performed on a Microsoft Surface 4 with 8gb RAM and a 2.2ghz Intel i7-6650U CPU.

While Scapetoad produced more accurate cartograms, SUNA was faster than Scapetoad by an order of magnitude. Using default parameters, computation time was approx. 1 second using SUNA for both default cartograms, whereas Scapetoad required 54 and 135 seconds for the U.S. and China cartograms, respectively. To achieve the higher accuracy shown in Fig. 8c-d, SUNA and Scapetoad required 11 and 1020 seconds, respectively.

Unfortunately, precise measurement of the time spent to construct the manual cartograms of the USA and China presented is not possible. This is because cartogram construction was performed simultaneously with continued development and improvement of the user interface. This iterative process was helpful from a software engineering perspective but dictates caution in any interpretation of time and effort spent, since the parameters of engagement were not constant. For example, a single adjustment might have required several minutes to compute at the beginning of the process, but the same adjustment might require only a fraction of a second at the end.

Nevertheless, some indication of the effort involved in manual cartogram construction can be determined from the number of adjustments performed. To facilitate analysis, each adjustment was recorded along with the prior and posterior apportionment errors. Fig. 13 shows the reduction in apportionment error over sequential adjustment for the manually constructed cartograms of the USA and China. For the USA, a sequence of 4,200 separate adjustments reduced an initial apportionment error of 38.4% down to 0.26%. This sequence is not complete, due to errors in record-keeping that occurred sometime after the 4,200th adjustment. However, it is estimated that several thousand additional adjustments were performed, in which apportionment error was reduced
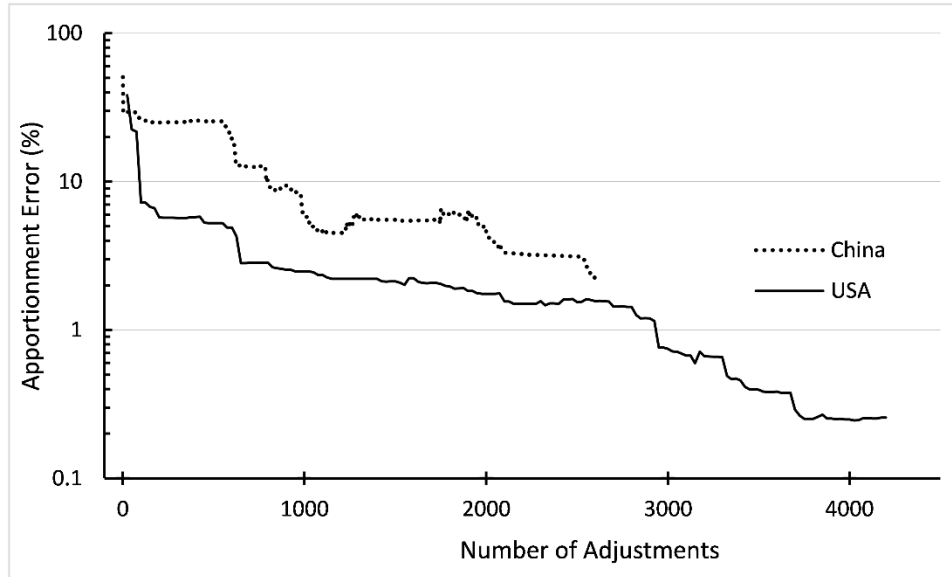
Figure 13: Decrease in apportionment error with manual adjustments in manually constructed cartograms of the USA and China.

only slightly further to 0.22%. The complete sequence for the cartogram of China comprises 2,599 adjustments that reduced apportionment error from 50.7% to 2.1%. Note that each "adjustment" represents a separate user action that typically involved simultaneous movement of multiple mesh vertices. For example, the 2,599 adjustments used to produce the China cartogram involved over 120,000 vertex movements in total.

Although the computation time required to implement each adjustment is typically well under one second, several seconds or more are required for the cognitive task of deciding what to adjust and how to adjust it. This does not include time spent designing and planning for the broader picture, or for adjustments that were tried and then reversed. Even if each adjustment represents just a few seconds of effort, this would add up to several hours to construct each cartogram. Thus, it can be concluded that the time required for manual cartogram construction must be measured in hours or days rather than minutes or seconds.

7.   Discussion

For decades, research on cartograms has focused on developing efficient and effective computer algorithms. Yet, at the same time it has been widely acknowledged that the primary obstacles to cartogram interpretation are cognitive and aesthetic. Given the overriding importance of aesthetics to this particular map form, perhaps it is time to question the nearly exclusive focus on algorithms in cartogram research. At this juncture, algorithmic approaches have developed to a far greater degree of sophistication than manual construction methods. As a result, it has become easier than ever to produce cartograms, but their aesthetic qualities continue to be questioned. Is it reasonable to expect algorithm research alone to identify optimal cartogram forms to support human cognitive function?

This paper explores the feasibility of using a joint triangulation framework to support manual cartogram construction, thus relying on a human cartographer to solve the primary aesthetic challenges in the hands of the human cartographer. The use of the joint triangulation framework is not without its drawbacks. Because user-defined transformations are applied to map districts indirectly through adjustment of the triangulation meshes, application is imprecise and involves greater computational burden. Indeed, the use of triangular meshes here runs counter to one of the

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key innovations of Dougenik et al. (1985), who found that removing the intermediary surface allowed much faster algorithm convergence and arguably better aesthetic quality. However, the use of a joint triangulation makes it easy to guarantee topological consistency and records a continuous transformation that can be applied post hoc to any dataset. Example cartograms of the United States and China show that the approach is feasible and can yield cartograms that are qualitatively different from cartograms produced algorithmically. Furthermore, and contrary to expectations, cartogram accuracy does not need to be sacrificed in the process.

The accuracy of manually constructed cartograms may not seem surprising given the large number of degrees of freedom afforded by the triangular meshes. However, algorithms also enjoy excess degrees of freedom, but the diffusion and carto3f algorithms were unable to match the accuracy of the manual cartogram of the United States. In addition, the algorithms consistently had trouble enlarging small but densely populated regions such as Washington D.C., Hong Kong and Macau. The problem was recognized by Sun (2013b), who noted that enlargement using an exponential distance-decay force generating function (used in the rubber-sheet algorithm) is limited to a factor of four at each iteration. The carto3f algorithm mitigates this through an enlargement exaggeration technique, but finding the optimal exaggeration parameter proved to be difficult. The joint triangulation technique suggested another approach that was quite effective, involving repositioning vertices on the source map rather than on the cartogram (Fig. 3). On the other hand, shrinking regions that were too large was relatively challenging using this technique, as evidenced by the error associated with Tibet and Inner Mongolia in the China cartogram (Fig. 11). The difficulty of handling sharp gradients in population density has been recognized before (Dougenik et al. 1985), but the asymmetry in this problem suggests that different techniques might be useful in handling densely and sparsely populated districts.

Though cartograms require shape deformation, there was not a strict inverse relationship between accuracy and aesthetic quality. In many cases it was possible to improve accuracy and aesthetic quality simultaneously, especially once the broad outlines of the cartograms were formed. In hindsight, some of the distortion produced by current software is obviously an artifact of the algorithms themselves and not a necessary consequence of seeking the correct district areas. For example, it is not difficult to imagine that the jagged outlines of the western provinces of Xinjiang and Tibet on the algorithmically produced cartograms of China (Fig. 9) could be smoothed considerably without affecting overall size or shape of neighboring provinces. The islands of Taiwan and Hainan are topologically independent from other provinces, and so in theory could be resized arbitrarily without any shape distortion at all. Perhaps these observations may lead to insights into how to improve cartogram algorithms.

Although the present research focused on manual cartogram construction, the joint triangulation framework could be used to integrate manual and algorithmic techniques. At least two possibilities are suggested. First, an automated algorithm could be used to create a "draft" cartogram which is then refined manually. If the algorithmic transformation is applied to an input triangular mesh, then the input and transformed meshes would form a joint triangulation that could serve as the basis for further manual refinement. A second possibility is to develop tools within the manual construction environment to apply algorithms locally to specific regions, providing the benefits of algorithmic efficiency but with finer user control. Further research in this area is warranted.

Given the advantages in aesthetic control and accuracy, the primary reason not to construct cartograms by hand is convenience. The time required to produce the manual cartograms presented here was two or three orders of magnitude greater than that required for the algorithmically produced cartograms. This will be a deterrent for many casual cartographers, and support for manual construction will not replace automated techniques that allow cartograms to be produced at the push of a button. However, a couple of observations are warranted.

First, manual cartograms can be constructed much more quickly if aesthetic concerns are ignored. That is, most of the time spent in manual construction went towards aesthetic refinement,

not reduction in apportionment error. This can be seen in the "step" pattern apparent in the graph of apportionment error over time (Fig. 13), which reveals two distinct phases of cartogram construction. The steep parts of this graph represent periods of effort spent focused on error reduction. These are separated by much longer phases of relative small improvements, during which the cognitive focus was on improving cartogram appearance. Overall for both cartograms, 90% of the reduction in apportionment error was accomplished with fewer than 5% of the manual adjustments.

Second, the extra effor required for manual cartogram construction might be engaging, especially in an educational setting. This sentiment echoes Dent (1972), who after having students construct cartograms by hand, commented that they not only learned a lot but also enjoyed the creative process. The task is challenging, to be sure, but the challenge has a puzzle-like quality as one searches for a configuration that is both aesthetically pleasing and reduces cartogram error.

At the same time, it is acknowledge that manual cartogram construction requires considerable time and effort under the proposed framework. It involves countless choices to be made by the cartographer regarding the appropriate shapes of individual districts and boundaries. Developing broader design objectives requires mental visualization and repeated trial and error. Inevitably, compromises must be made between competing objectives, and sometimes an initial design choice will turn out to be infeasible when other goals are considered. However, it is precisely this process of engagement that holds promise for improving cartograms as a map form. By grappling mentally with the infinite geometric possibilities, manual construction efforts will not only produce better cartograms, but also help us to understand what makes a cartogram effective, and thereby also lead to better algorithms.

8.   Conclusions

This article explores the feasibility of using a mesh transformation framework to facilitate manual construction of cartograms. The framework guarantees topological preservation of vector geographic datasets with insertion of a finite number of points. When vertices are repositioned on the source mesh, the shape distortion effects of enlarging small, densely populated districts can be contained within a small neighborhood. Thus, construction of topologically correct and numerically accurate cartograms is supported. A prototype application based on this framework proved capable of supporting construction of accurate cartograms with distinct aesthetic qualities. Accurate cartograms of the conterminous United States and China were constructed featuring straight lines, broad curves and emphasis on cognitively salient landmarks.

The proposed framework offers an alternative to the paradigm of algorithmic construction that has dominated cartogram research for several decades. In addition to facilitating the construction of individually effective and innovative cartograms, manual construction should serve a number of broader purposes in cartogram research. By enabling freeform exploration of a variety of cartogram forms, comparative cognitive research into cartogram use and effectiveness can be supported. Experimentation with manual cartogram construction may also suggest approaches to improve current algorithms. Finally, education in the purpose and use of cartograms might be enhanced by the creative process involved in their manual construction.

9.   Acknowledgements

10.   References

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