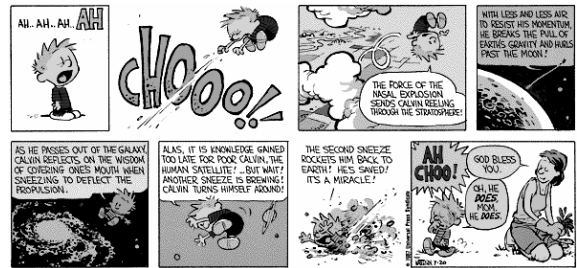


## Momentum

## Calvin & Hobbes by Bill Watterson



## Newton's 2<sup>nd</sup> Law Reprise

- Newton's second law  $\vec{F}_{net} = m \cdot \vec{a}$

- can be written:  $\vec{F}_{net} = m \frac{\Delta \vec{v}}{\Delta t}$

$$\vec{F}_{net} \cdot \Delta t = m \cdot \Delta \vec{v}$$

$$\text{Impulse} = \vec{F}_{net} \cdot \Delta t$$

$$\text{Change in momentum} = m \cdot \Delta \vec{v}$$

Impulse = Change in momentum

## Momentum & Impulse

- Momentum:  $\vec{p}$ ; kg-m/s
- Impulse:  $\vec{J}$ ; N-s
- Show that N-s are equivalent to kg-m/s

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$$N \cdot s = kg \cdot \frac{m}{s^2} \cdot s = kg \cdot \frac{m}{s}$$

## Golf Ball Momentum

- Mass of golf ball = 0.0459 kg
- Speed of golf ball leaving tee on drive = 70 m/s
- What is the momentum of the golf ball?
- If the golf club is in contact with the ball for 0.5 ms, what is the average force exerted on the ball by the club?



### Golf Ball Momentum

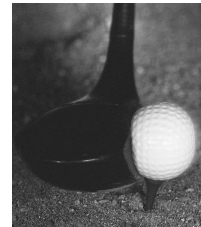
- Mass of golf ball = 0.0459 kg
- Speed of golf ball leaving tee on drive = 70 m/s



- momentum = (mass)(velocity)
- $P = (0.0459 \text{ kg})(70 \text{ m/s}) = 3.21 \text{ kg}\cdot\text{m/s}$

### Force on Golf Ball

- Mass of golf ball = 0.0459 kg
- Speed of golf ball leaving tee on drive = 70 m/s



- $P = 3.21 \text{ kg}\cdot\text{m/s}$
- $\Delta P = 3.21 \text{ kg}\cdot\text{m/s} - 0 = 3.21 \text{ kg}\cdot\text{m/s}$
- If the golf club is in contact with the ball for 0.5 ms, what is the average force exerted on the ball by the club?

$$\vec{F}_{net} \cdot \Delta t = m \cdot \Delta \vec{v} \quad \vec{F}_{net} = \frac{3.21 \text{ kg} \cdot \text{m/s}}{0.0005 \text{ s}} = 6,420 \text{ N}$$

$$\vec{F}_{net} \cdot 0.0005 \text{ s} = 3.21 \text{ kg} \cdot \text{m/s}$$

### Decreasing Momentum Over a Long Time

- Hitting dashboard compared to air bag
- Landing stiff-legged compared to bending knees
- Wooden floor compared to tile
- Catching a baseball

$$\vec{F}_{net} \cdot \Delta t = \Delta \vec{P}$$

- $\Delta P$  is fixed value
- To minimize F, increase  $\Delta t$

### Bouncing

- Which undergoes the greatest change in momentum: (1) a baseball that is caught, (2) a baseball that is thrown, or (3) a baseball that is caught and then thrown back, if the baseballs have the same speed just before being caught and just after being thrown?

### Bouncing

- Which undergoes the greatest change in momentum: (1) a baseball that is caught, (2) a baseball that is thrown, or (3) a baseball that is caught and then thrown back, if the baseballs have the same speed just before being caught and just after being thrown?
- (3) because it has twice the momentum change of either (1) or (2)

### Momentum Conservation

- Newton's second law can be written:

$$\vec{F}_{EXT} = \frac{\Delta \vec{P}}{\Delta t}$$

- It tells us that if  $\vec{F}_{EXT} = 0$ , the total momentum of the system does not change.

← The total momentum of a system is conserved if there are no external forces acting.

← In physics, when we say a quantity is conserved, we mean that it remains constant.

### Momentum Conservation

$$F_{EXT} = \frac{\Delta P}{\Delta t} \Rightarrow \frac{\Delta P}{\Delta t} = 0 \Leftarrow F_{EXT} = 0$$

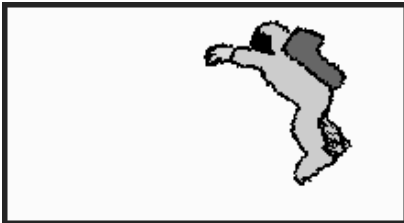
- The concept of momentum conservation is one of the most fundamental principles in physics.
- This is a component (vector) equation.
  - ◀ We can apply it to any direction in which there is no external force applied.
- You will see that we often have momentum conservation even when energy is not conserved.

### Momentum Conservation



- if mass of each astronaut is 60 kg, and the astronaut on the left is initially moving at 5 m/s, how fast does the pair move after the collision?

### Momentum Conservation



- if mass of each astronaut is 60 kg, and the astronaut on the left is initially moving at 5 m/s, how fast does the pair move after the collision?
- $mv = (2m)v'$
- $(60 \text{ kg}) \cdot (5 \text{ m/s}) = (120 \text{ kg}) \cdot v'$
- $v' = (300/120) = 2.5 \text{ m/s}$

### Big Fish Little Fish

The mass of the big fish is 4X the mass of the little fish.  
Speed of Big Fish = 5 km/hr



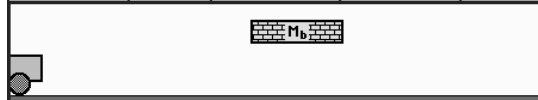
### Big Fish Little fish 2

The mass of the big fish is 4X the mass of the little fish.  
Speed of Small Fish = 5 km/hr



### Brick on Cart

Cart		Dropped Brick	
Mass (kg)	1.0	Mass (kg)	2.0
Vel. (cm/s)	60.0	Vel. (cm/s)	0.0
Mom. (kg cm/s)	60.0	Mom. (kg cm/s)	0



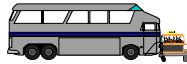
### Elastic vs. Inelastic Collisions

- A collision is said to be *elastic* when colliding objects rebound without lasting deformation or generation of heat.
- Carts colliding with a spring in between, billiard balls, etc.



- A collision is said to be *inelastic* when the colliding objects become distorted, generate heat, and possibly stick together

← Car crashes, collisions where objects stick together, etc.



### Momentum Conservation

- Two balls of equal mass are thrown horizontally with the same initial velocity. They hit identical stationary boxes resting on a frictionless horizontal surface.
- The ball hitting box 1 bounces back, while the ball hitting box 2 gets stuck.

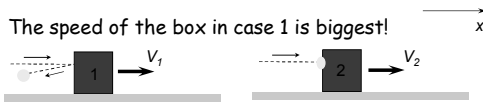
← Which box ends up moving faster?

- (a) Box 1      (b) Box 2      (c) same



### Momentum Conservation

- Since the total external force in the x-direction is zero, momentum is conserved along the x-axis.
- In both cases the initial momentum is the same ( $mv$  of ball).
- In case 1 the ball has negative momentum after the collision, hence the box must have more positive momentum if the total is to be conserved.
- The speed of the box in case 1 is biggest!



### Momentum Conservation

$$mv_{\text{init}} = MV_1 - mv_{\text{fin}} \quad \Rightarrow \quad V_1 = (mv_{\text{init}} + mv_{\text{fin}}) / M$$

$$mv_{\text{init}} = (M+m)V_2 \quad \Rightarrow \quad V_2 = mv_{\text{init}} / (M+m)$$

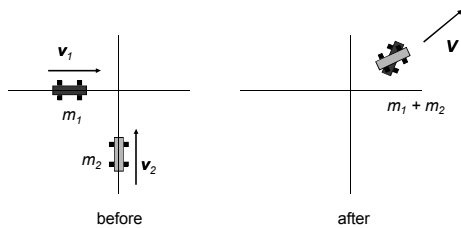
$V_1$  numerator is bigger and its denominator is smaller than that of  $V_2$ .

$$\Rightarrow V_1 > V_2$$



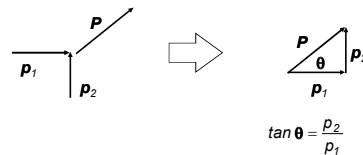
### Inelastic collision in 2-D

- Consider a collision in 2-D (cars crashing at a slippery intersection...no friction).

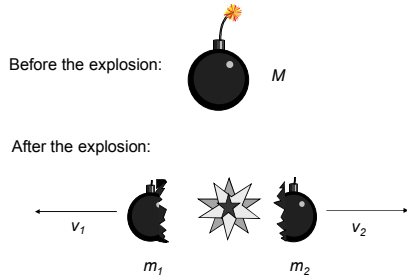


### Inelastic collision in 2-D...

- We can see the same thing using vectors:



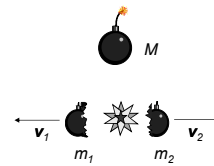
### Explosion (inelastic un-collision)



### Explosion...

- No external forces, so  $P$  is conserved.
- Initially:  $P = 0$
- Finally:  $P = m_1v_1 + m_2v_2 = 0$

$$m_1v_1 = -m_2v_2$$



5. Granny whizzes around the rink and is suddenly confronted with Ambrose at rest directly in her path. Rather than knock him over, she picks him up and continues in motion without "baking". Consider both Granny and Ambrose as two parts of one system. Since no outside forces act on the system, the momentum of the system before collision equals the momentum of the system after collision.

a. Complete the before-collision data in the table below.

BEFORE COLLISION	
Granny's mass	80 kg
Granny's speed	3 m/s
Granny's momentum	
Ambrose's mass	40 kg
Ambrose's speed	0 m/s
Ambrose's momentum	
Total momentum	

b. After collision, Granny's speed [increases] [decreases]

c. After collision, Ambrose's speed [increases] [decreases]

d. After collision, the total mass of Granny + Ambrose is \_\_\_\_\_

e. After collision, the total momentum of Granny + Ambrose is \_\_\_\_\_

f. Use the conservation of momentum law to find the speed of Granny and Ambrose together after collision. (Show your work in the space below.)

Now speed: \_\_\_\_\_

