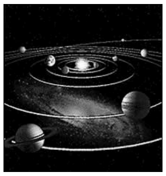
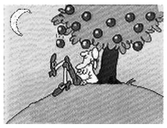
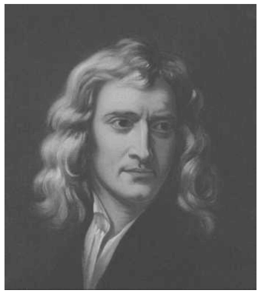
Universal Gravitation



Celestial



Terrestrial



Sir Isaac Newton

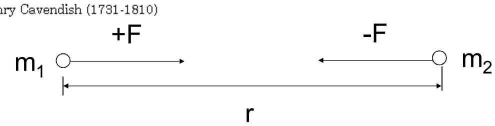
1642-1727

UNIVERSAL GRAVITATION

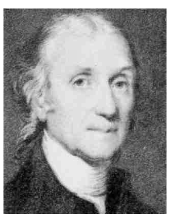
For any two masses in the
se: $F = G\, m_1 m_2 / r^2$

Henry Cavendish (1731-1810)

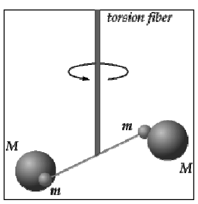
G = a constant later evaluated by Cavendish



CAVENDISH: MEASURED G

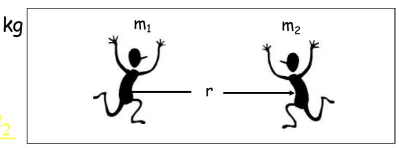


Modern value:
$G = 6.674 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$



Two people pass in a hall. Find the gravitational force between them.

- $m_1 = m_2 = 70$ kg
- $r = 1$ m



$$F = G\frac{m_1 \cdot m_2}{r^2}$$

$F = (6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2)(70\ \text{kg})(70\ \text{kg})/(1\ \text{m})^2$

$F = 3.3 \times 10^{-7}$ N

Earth-Moon Force

- Mass of Earth: 5.97×10^{24} kg
- Mass of Moon: 7.35×10^{22} kg
- Earth-Moon Distance: 3.84×10^{8} m

- What is the force between the earth and the moon?

$F = (6.67 \times 10^{-11}\ \text{N m}^2/\text{kg}^2)(5.97\times10^{24}\text{kg})(7.35\times10^{22})/(3.84\times10^{8})^2$

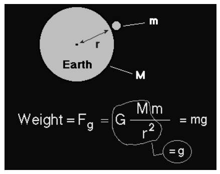
1.98×10^{20} N

Practice

- What is the gravitational force of attraction between a 100 kg football player on the earth and the earth?

Definition of Weight

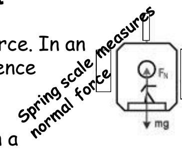
- The weight of an object is the gravitational force the earth exerts on the object.
  - ←Weight = $GM_E m/R_E^2$
- Weight can also be expressed
  - ←Weight = mg
- Combining these expressions
  - ←$mg = GM_E m/R_E^2$
    - » $R_E = 6.37 \times 10^6$ m = 6370 km
    - » $M_E = 5.97 \times 10^{24}$ kg
  - ←$g = GM_E/R_E^2 = 9.8$ m/s^2
- The value of the gravitational field strength (g) on any celestial body can be determined by using the above formula.



Apparent Weight

Apparent Weight is the normal support force. In an inertial (non-accelerating) frame of reference

- $F_N = F_G$

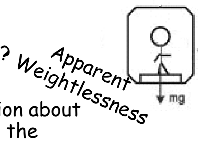


What is the weight of a 70 kg astronaut in a satellite with an orbital radius of 1.3×10^7 m?

Weight = GMm/r^2 Using: $G = 6.67 \times 10^{-11}$ N-m^2/kg^2

and M = 5.98×10^{24} kg Weight = 165 N

What is the astronaut's apparent weight?

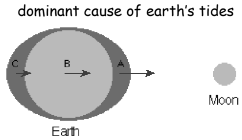


The astronaut is in uniform circular motion about Earth. The net force on the astronaut is the gravitational force. The normal force is 0. The astronaut's apparent weight is 0.

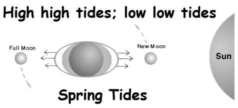
Tides

- $F_{G \text{ by moon on } A} > F_{G \text{ by moon on } B}$
- $F_{G \text{ by moon on } B} > F_{G \text{ by moon on } C}$
- Earth-Moon distance: 385,000 km which is about 60 earth radii
- Sun also produces tides, but it is a smaller effect due to greater Earth-Sun distance.
  - ←1.5×10^8 km

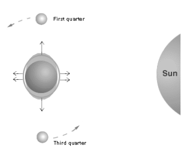
Different distances to moon is dominant cause of earth's tides



High high tides; low low tides



Spring Tides



Low high tides; high low tides

Neap Tides

Tide Animation

- http://www.youtube.com/watch?v=Ead8d9wVDTQ

Satellite Motion

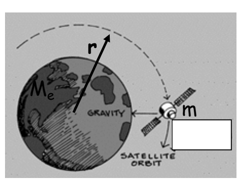
The net force on the satellite is the gravitational force.

$F_{net} = F_G$

Assuming a circular orbit:

$ma_c = GmM_e/r^2$

$$\frac{mv^2}{r} = G\frac{mM_e}{r^2}$$

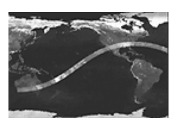


$$v = \sqrt{\frac{GM_e}{r}}$$

Note that the satellite mass cancels out.

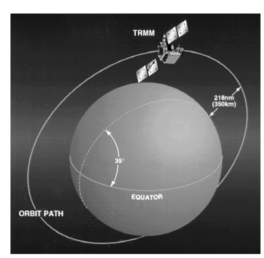
Using $M_e = 5.97 \times 10^{24}\,kg$

For low orbits (few hundred km up) this turns out to be about 8 km/s = 17000 mph



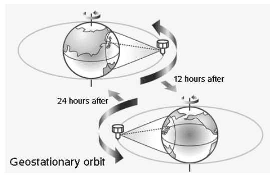
TRMM
Tropical Rainfall Measuring Mission

- The TRMM orbit is circular and is at an altitude of 218 nautical miles (350 km) and an inclination of 35 degrees to the Equator.
- The spacecraft takes about 91 minutes to complete one orbit around the Earth. This orbit allows for as much coverage of the tropics and extraction of rainfall data over the 24-hour day (16 orbits) as possible.



Geosynchronous Satellite

In order to remain above the same point on the surface of the earth, what must be the period of the satellite's orbit? What orbital radius is required?



T = 24 hr = 86,400 s

$$F_{net} = F_G$$

$$\frac{mv^2}{r} = G\frac{mM_e}{r^2}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM_e}{r}$$

$$r^3 = \frac{GM_e T^2}{4\pi^2}$$

Actually the theoretical derivation of Kepler's Third Law

Using $M_e = 5.97\times10^{24}\ kg$

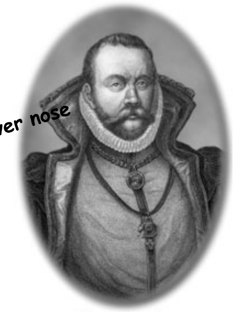
r = 42,000 km = 26,000 mi

A Colorful Character

- Highly accurate data
- Gave his data to Kepler

Lost nose in a duel

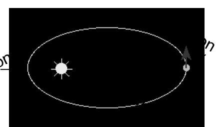
Copper/silver nose

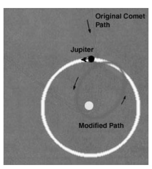


Tycho Brahe
(1546-1601)

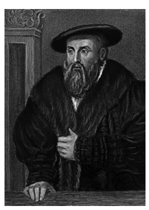
Kepler's First Law

- The orbit of a planet/comet about the Sun is an ellipse with the Sun's center of mass at one focus



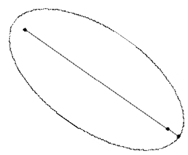


A comet falls into a small elliptical orbit after a "brush" with Jupiter



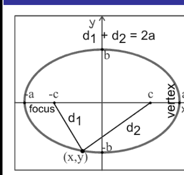
Johannes Kepler
1571-1630

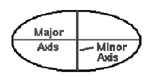
$PF_1 + PF_2 = 2a$



Orbital Eccentricities

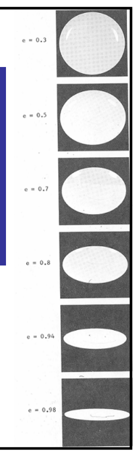
Planet	Eccentricity	Notes
Mercury	0.206	Too few observations for Kepler to study
Venus	0.007	Nearly circular orbit
Earth	0.017	Small eccentricity
Mars	0.093	Largest eccentricity among planets Kepler could study
Jupiter	0.048	Slow moving in the sky
Saturn	0.056	Slow moving in the sky
Uranus	0.470	Not discovered until 1781
Neptune	0.009	Not discovered until 1846
Pluto	0.249	Not discovered until 1930





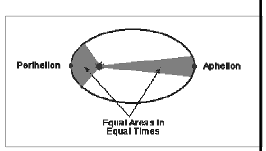
eccentricity = c/a

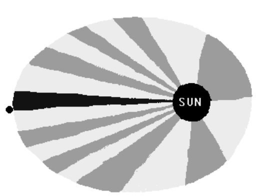
or distance between foci divided by length of major axis



Kepler's Second Law

- Law of Equal Areas
- A line joining a planet/comet and the Sun sweeps out equal areas in equal intervals of time





$$\frac{v_p}{v_a} = \frac{R_a}{R_p}$$

Kepler's Third Law

Square of any planet's orbital period (sidereal) is proportional to cube of its mean distance (semi-major axis) from Sun

$T^2 = K R_{av}^3$ $R_{av} = (R_a + R_p)/2$

Recall from a previous slide the derivation of $T^2 = [4\pi^2/GM]r^3$ from $F_{net} = F_G$

$K = 4\pi^2/GM$

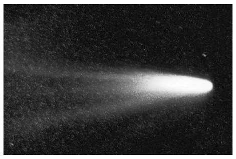
Planet	T (yr)	R (AU)	T^2	R^3
Mercury	0.24	0.39	0.06	0.06
Venus	0.62	0.72	0.39	0.37
Earth	1.00	1.00	1.00	1.00
Mars	1.88	1.52	3.53	3.51
Jupiter	11.9	5.20	142	141
Saturn	29.5	9.54	870	868

K for our sun as the primary is 1 yr^2/AU^3

The value of K for an orbital system depends on the mass of the primary

HALLEY'S COMET

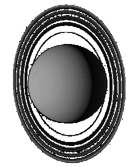
He observed it in 1682, predicting that, if it obeyed Kepler's laws, it would return in 1759.



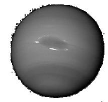
When it did, (after Halley's death) it was regarded as a triumph of Newton's laws.

DISCOVERY OF NEW PLANETS

Small departures from elliptical orbits occur due to the gravitational forces of other planets.

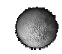


Deviations in the orbit of Uranus led two astronomers to predict the position of another unobserved planet.



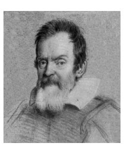
This is how Neptune was added to the Solar System in 1846.

Deviations in the orbits of Uranus and Neptune led to the discovery of Pluto in 1930

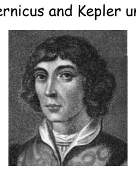


Newton
Universal Gravitation

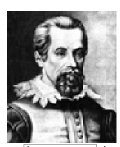
- Three laws of motion and law of gravitation
- eccentric orbits of comets
  - ← cause of tides and their variations
  - ← the precession of the earth's axis
  - ← the perturbation of the motion of the moon by gravity of the sun
- Solved most known problems of astronomy and terrestrial physics
  - ← Work of Galileo, Copernicus and Kepler unified.



Galileo Galili
1564-1642



Nicholaus Copernicus
1473-1543



Johannes Kepler
1571-1630

Simulations & Videos

- http://www.cuug.ab.ca/kmcclary/
- http://www.youtube.com/watch?v=fxwjeg_r5Ug
- http://www.youtube.com/watch?v=AAqSCuHA0j8
- http://www.youtube.com/watch?v=0rocNtnD-yI