when continuum varies, broad line respond within ≤ 1 month $\Rightarrow R = ct = 10^{15}$ m $\Rightarrow$ relatively close to center.

Unified model (Fig. 28.25) shows how different viewing angles could give various observed AGN types.

Example: Seyfert 2s have narrow lines only because broad line region is obscured by torus.

Blazar produced when jet points towards observer.

p. 1117 Determining BH Masses ~~in Broad Line Regions~~

Broad emission lines $\Rightarrow$ clouds orbit.

Width of line $= 5000$ km s^{-1} = orbital velocity, then $r = 10^{15}$ m $\Rightarrow M_{bh} = \frac{rv^2}{G} = 1.9 \times 10^8 M_\odot$

Time delay from continuum brightness changes to line changes $\Rightarrow$ reverberation mapping. $\Rightarrow$ (somehow) M_{BH} (Fig. 28.26) (lots of uncertainty)

p. 1118 The Narrow-Line Region

(Fig. 28.25) The narrow-line region seems to be a clumpy ~spherical distribution of gas w/ $T \sim 10^4$ K

Ex. 28.2.2 (skip) estimates the filling factor to be ~ 0.02 - clouds occupy 2% of volume.

(Fig. 28.27) Explain caption. Flow could be driven by radiation pressure, wind from accretion disk, or could be part of jet.

p. 1121 A Summary of the Unified Model of AGNs

Central engine is accretion disk orbiting rotating SMBH.

Powered by conversion of grav. P.E. into synchrotron rad., & maybe also rotational K.E. of BH.

$\dot{M} = 1\text{-}10\ M_\odot\ \text{yr}^{-1}$.

Observation angle, $\dot{M}$, & M_{SMBH} determine whether AGN is Seyfert 1 or 2, BLRG (broad-line radio galaxy), NLRG, or radio-loud vs. quiet quasar.

(Fig. 28.28) HST image of elliptical radio galaxy NGC 4261.

$M_{SMBH} \sim 10^7 M_\odot$, $r_{torus} \sim 70$ pc, jets extend to 15 kpc.

($R_s \sim 3 \times 10^{10}$ m < 1 nm on photo).

Ch. 29 Cosmology

§29.1 Newtonian Cosmology p. 1144

Cosmology is the study of the origin & evolution of the universe.

In this section we will develop our intuition by using Newtonian physics, w/o GR or particle physics.

Olber's Paradox p. 1145

Newton believed in an infinite static universe filled uniformly w/ stars.

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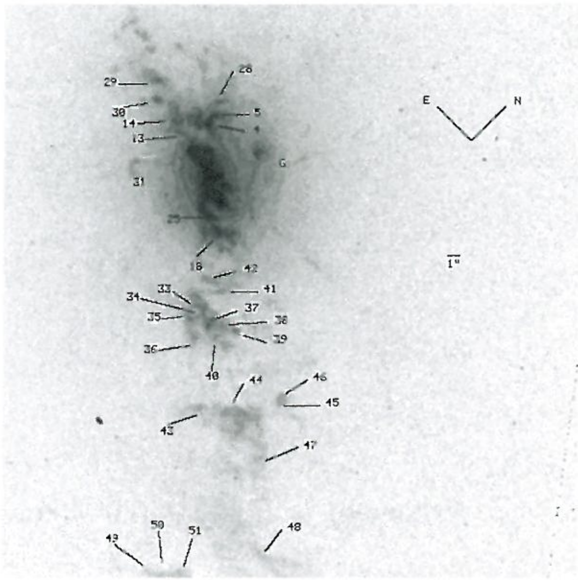

Fig. 28.27 An HST image of the narrow-line region of the Seyfert 1 galaxy NGC 4151. Numerous clouds are evident in a biconical distribution. The clouds in the SW are approaching the observer relative to the nucleus, & the clouds in the NE have recessional velocities. There is some evidence that the clouds may be associated with the galaxy's radio jets.

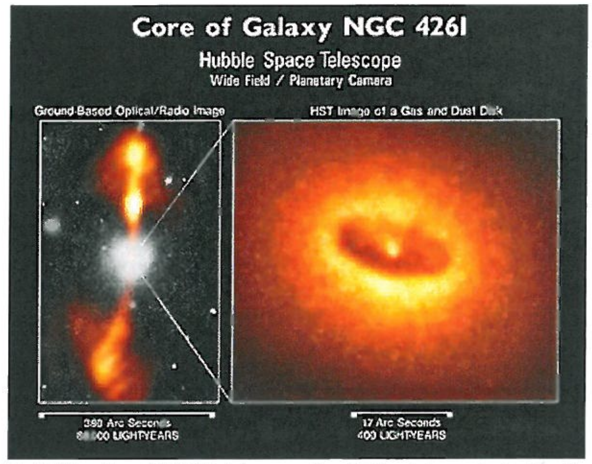

Fig. 28.28 Two views of NGC 4261. Left: a composite optical & radio image, showing the radio jets. Right: an optical image from HST, showing the dusty torus around the nucleus.

§28.3 starts here

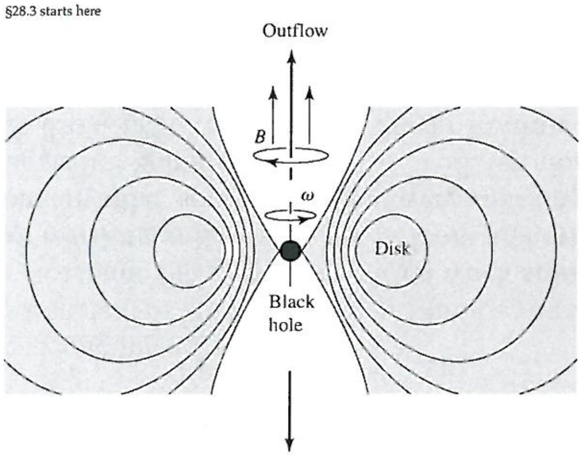

Fig. 28.29

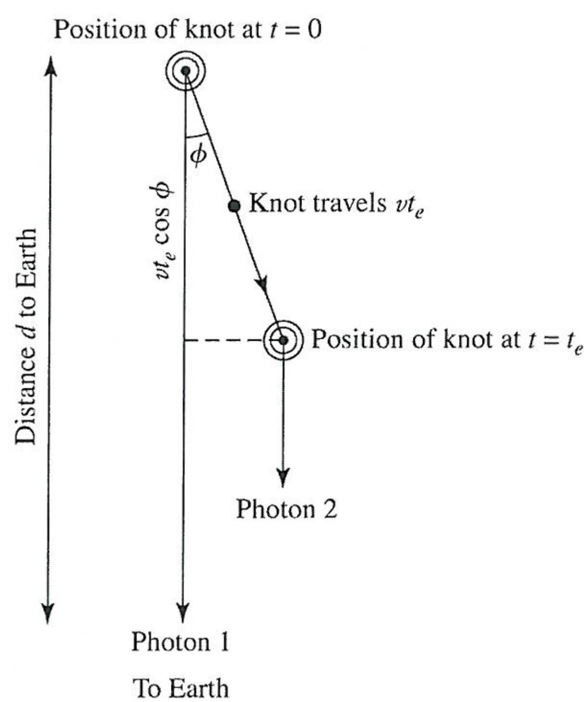

Fig. 28.32

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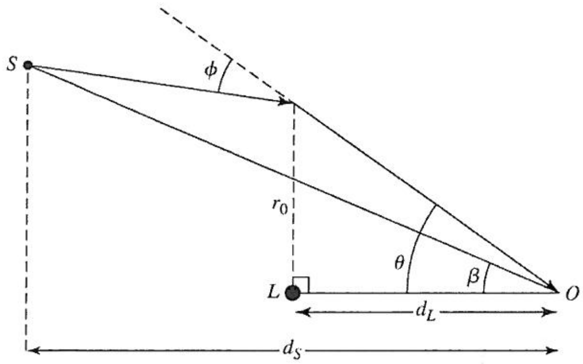

Fig. 28.35

But why is the night sky dark?
Olber's paradox - after German physician Heinrich Olbers (1758–1840).
Every line of sight should hit a star, like trees in an infinite forest.
Olbers thought solution was that space not transparent, but obscuring matter would heat up till as bright as star.
Resolved by Edgar Allen Poe (1809-49) - light has finite speed + universe not infinitely old ⇒ light from distant sources not arrived yet.

Developing our Intuition: A Newtonian Approach

The text warns us that there's a lot of math in this section.

The Cosmological Principle p.1145

= the assumption that the universe is isotropic + homogeneous - the same in all directions + locations.

(Fig. 29.1) Earth at origin, galaxies at A + B.
Hubble law $\vec{v}_A = H_0 \vec{r}_A$, $\vec{v}_B = H_0 \vec{r}_B \Rightarrow \vec{v}_B - \vec{v}_A = H_0(\vec{r}_B - \vec{r}_A) \Rightarrow$ Hubble law also applies from Galaxy A (or anywhere).
Actually $H = H(t)$, $H_0 = H(t_0)$.

A Simple Pressureless "Dust" Model of the Universe p.1146

Single-component model of universe - pressureless homogeneous "dust".
Examine spherical shell in expanding universe. (Fig. 29.2)
Cons. of E for this shell: $E = K(t) + U(t) = \frac{1}{2} m v^2(t) - G \frac{M_r m}{r(t)} = -\frac{1}{2} m k c^2 \varpi^2$ (29.1)
E written in terms of constants k + ϖ (varpi) aka "pomega".
k (units length^{-2}) discussed in §29.3
ϖ labels this shell, can be thought of as present radius of shell: $r(t_0) = \varpi$.
M_r = mass interior to shell $= \frac{4}{3}\pi r^3(t)\rho(t)$
(29.1) $\Rightarrow v^2 - \frac{8}{3}\pi G \rho r^2 = -kc^2\varpi^2$ (29.2)
k determines fate of universe
$k > 0 \Rightarrow E < 0$, universe bounded = closed, expansion will stop + reverse.
$k < 0 \Rightarrow E > 0$, " unbounded = open, " continues forever.
$k = 0 \Rightarrow E = 0$, " flat, $v \to 0$ as $t \to \infty$.
Newtonian spacetime is flat. In §29.3, closed, open, + flat will refer to curvature of spacetime.
$r(t)$ = coordinate distance,
ϖ = comoving coordinate (Fig. 29.3) $r(t) = R(t)\varpi$ (29.3)
$R(t)$ = dimensionless scale factor, same for all shells.
$R(t_0) = 1 \Rightarrow r(t_0) = \varpi$
$R = \frac{R_{emitted}}{R_{obs}} = \frac{1}{1+z}$ (29.4)
M_r = const. $\Rightarrow \rho r^3$ = const. $\Rightarrow R^3(t)\rho(t) = R^3(t_0)\rho(t_0) = \rho_0$ (since $R(t_0) = 1$) (29.5)
$\rho = \rho_0 / R^3 = \rho_0(1+z)^3$ (29.6)

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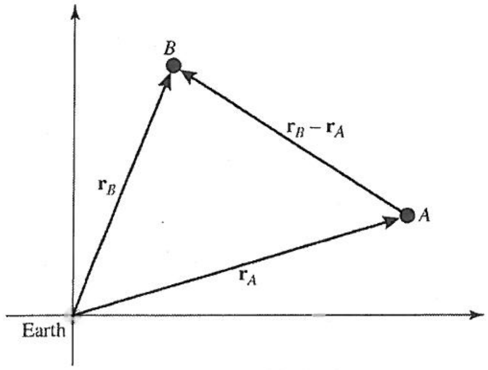

Fig. 29.1 The expansion of the universe, with Earth at the origin.

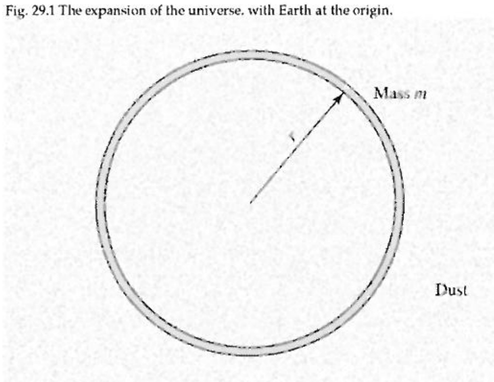

Fig. 29.2 Spherical mass shell in a dust-filled universe.

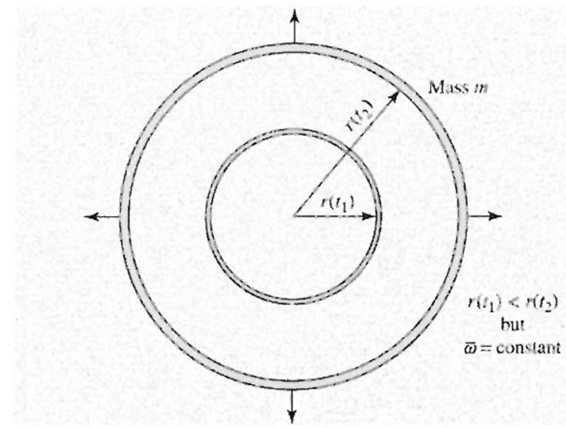

Fig. 29.3 An expanding mass shell seen at 2 different times, $t_1 < t_2$.

Method	M/L ($M_\odot/L_\odot$)	Ω_0
Solar neighborhood	3	$0.002h^{-1}$
Elliptical galaxy cores	$12h$	0.007
Local escape speed	30	$0.018h^{-1}$
Satellite galaxies	30	$0.018h^{-1}$
Magellanic Stream	> 80	$> 0.05h^{-1}$
X-ray halo of M87	> 750	$> 0.46h^{-1}$
Local Group timing	100	$0.06h^{-1}$
Groups of galaxies	$260h$	0.16
Clusters of galaxies	$400h$	0.25
Gravitational lenses	—	$0.1 - 0.3$
Big Bang nucleosynthesis	—	0.065 ± 0.045

Table 29.1 Mass-to-light ratios & density parameters, measured for a variety of systems.

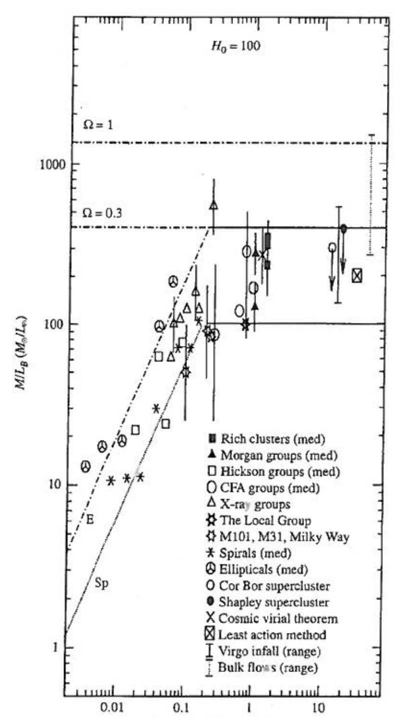

Fig. 29.4 The mass-to-light ratio as a function of the characteristic size of a variety of systems, using $H_0 = 100$ km s^{-1} Mpc^{-1} (pre-WMAP).

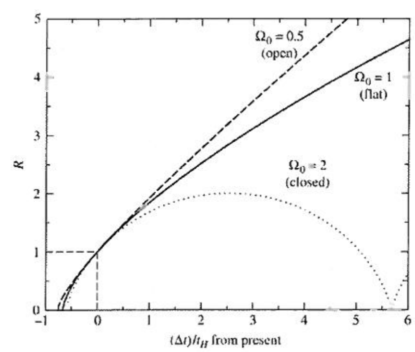

Fig. 29.5 The evolution of the scale factor, R, for 3 model universes. The dotted lines locate the position of today's universe. At the present (R = 1) all 3 universes have the same value of H_0, as exhibited by the curves having the same slope.

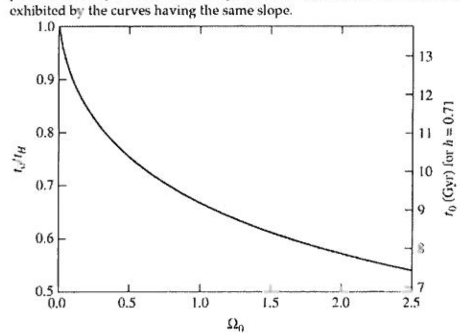

Fig. 29.6 The age of the universe as a function of the density parameter, Ω_0. The age is expressed as a fraction of the Hubble time, $t_H \simeq 10^{10} h^{-1}$ yr. The right axis shows the age in billions of years for $h = 0.71$.

4/8/16, (ρ contains u/c^2 component, which changes!)

The previous 2 eqs. are valid only for pressureless dust universe.

The Evolution of the Pressureless "Dust" Universe p. 1149

$v(t) = H(t)r(t) = H(t)R(t)\varpi = \frac{dr(t)}{dt} = \frac{d}{dt}(R(t)\varpi) = \frac{dR(t)}{dt}\varpi \Rightarrow$

$H(t) = \frac{1}{R(t)}\frac{dR(t)}{dt}$ (29.8)

$29.2 \Rightarrow H^2R^2\varpi^2 - \frac{8}{3}\pi G\rho R^2\varpi^2 = -kc^2\varpi^2 \Rightarrow (H^2 - \frac{8}{3}\pi G\rho)R^2 = -kc^2$ (29.9)

$29.8 \Rightarrow \left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2$ (29.10)

$29.5 \Rightarrow \left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_0}{3R} = -kc^2$ (29.11)

flat universe $k=0$: $(29.9) \Rightarrow \rho = \rho_c =$ critical density $= \frac{3H^2(t)}{8\pi G}$

current value $\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 1.88\times10^{-26}h^2$ kg m^{-3}

For $h = h_{WMAP} = 0.71$, this is $\rho_{c,0} = 9.47\times10^{-27}$ kg m^{-3} $\simeq 6\ \frac{\text{H atoms}}{\text{m}^3}$

WMAP value for baryonic matter is $\rho_{b,0} = 4.17\times10^{-28}$ kg m^{-3} $\simeq 0.04\,\rho_{c,0}$

"Baryonic" actually includes any particles satisfying (29.5), but not γ's, ν's.

Our pressureless "dust" universe includes baryonic + nonbaryonic, luminous + dark.

Define density parameter $\Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)} = \frac{8\pi G\rho(t)}{3H^2(t)}$ (29.18)

with current value $\Omega_0 = \frac{\rho_0}{\rho_{c,0}} = \frac{8\pi G\rho_0}{3H_0^2}$ (29.19)

(Table 29.1) M/L + Ω's for astronomical systems.

(Fig. 29.4) Largest systems $\Rightarrow$ "ceiling" at $\Omega_0 \simeq 0.3$

For all matter (baryonic + dark) WMAP gives $[\Omega_{m,0}]_{WMAP} = 0.27 \pm 0.04$

For baryonic matter, $[\Omega_{b,0}]_{WMAP} = 0.044 \pm 0.004 \simeq 16\%$ of $\Omega_{m,0}$
(the rest is dark matter).

$(29.6 + 19) \Rightarrow \frac{\Omega}{\Omega_0} = \frac{\rho}{\rho_0}\frac{H_0^2}{H^2} = (1+z)^3\frac{H_0^2}{H^2}$ (29.23)

$(29.18 + 9) \Rightarrow -kc^2 = (H^2 - H^2\Omega)R^2 = H^2(1-\Omega)R^2$ (29.24)

At t_0 this is $H_0^2(1-\Omega_0) = -kc^2$ (29.25) $\Rightarrow$

$\Omega_0 > 1 \Rightarrow k > 0$ (closed)

$\Omega_0 < 1 \Rightarrow k < 0$ (open)

$\Omega_0 = 1 \Rightarrow k = 0$ (flat)

This is true only for our one-component pressureless "dust" universe.

$(29.24, 25, + 4) \Rightarrow H^2(1-\Omega) = -kc^2/R^2 = H_0^2(1-\Omega_0)(1+z)^2$ (29.26)

$(29.23, 26) \cdots \Rightarrow H = H_0(1+z)(1+\Omega_0 z)^{\frac{1}{2}}$ (29.27)

& $\Omega = \left(\frac{1+z}{1+\Omega_0 z}\right)\Omega_0 = 1 + \frac{\Omega_0 - 1}{1+\Omega_0 z}$ (29.28)

(29.27) ⇒ at early times ($R \to 0$, $z \to \infty$), $H \to \infty$.

(29.28) ⇒ Ω does not change sign, + if $\Omega = 1$ at some time, $\Omega = 1$ forever

⇒ it is either always closed, open, or flat.

Also (29.28) ⇒ $\Omega \to 1$ as $z \to \infty$ ⇒ early universe was essentially flat. (Fig. 29.5)

Ex. 29.1.1 p. 1154 We will see that at $t = 3$ min., $p + n \to He$.

this occurred at $z = 3.68 \times 10^8$. Using $\Omega_0 = [\Omega_{m,0}]_{WMAP} = 0.27$ ⇒

$\Omega = 0.99999999265$ (8 9's)

Late 20th century ⇒ theoreticians couldn't see how there could be a mechanism to tune Ω this close to, but not exactly to 1.

Solution will be described in §29.3.

Expansion of flat dust universe: (29.11 w/ $\rho_0 = \rho_{c,0}$, $k=0$, $\Omega_0 = 1$

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G \rho_{c,0}}{3R} \Rightarrow R^{\frac{1}{2}} dR = \left(\frac{8\pi G \rho_{c,0}}{3}\right)^{\frac{1}{2}} dt$$

$$\int_0^R (R')^{\frac{1}{2}} dR' = \frac{2}{3} R^{\frac{3}{2}} = \left(\frac{8\pi G \rho_{c,0}}{3}\right)^{\frac{1}{2}} \int_0^t dt' = \left(\frac{8\pi G \rho_{c,0}}{3}\right)^{\frac{1}{2}} t \overset{29.15}{=} H_0 t$$

$$R_{flat} = (6\pi G \rho_{c,0})^{\frac{1}{3}} t^{\frac{2}{3}} = \left(\frac{3}{2}\right)^{\frac{2}{3}} \left(t/t_H\right)^{\frac{2}{3}}$$ (Fig. 29.5) (29.31)

For $\Omega_0 \neq 1$, (29.11) is more difficult to solve.

express solution in parametric form

$\Omega_0 > 1$ (closed)

$$R_{closed} = \frac{1}{2} \frac{\Omega_0}{\Omega_0 - 1} [1 - \cos x], \quad t_{closed} = \frac{1}{2H_0} \frac{\Omega_0}{(\Omega_0 - 1)^{\frac{3}{2}}} [x - \sin x]$$ (29.35)

(Fig. 29.5) "Bounce" is mathematical artifact.

$\Omega_0 < 1$ (open) ⇒

$$R_{open} = \frac{1}{2} \frac{\Omega_0}{1 - \Omega_0} [\cosh x - 1], \quad t_{open} = \frac{1}{2H_0} \frac{\Omega_0}{(1 - \Omega_0)^{\frac{3}{2}}} [\sinh x - x]$$ (29.39)

(Fig. 29.5)

The Age of the Pressureless "Dust" Universe p. 1156

"Age of universe" is an extrapolated time, known laws of physics break down for $t < 10^{-43}$ s. There might have even been a previous universe.

For flat universe, w/ $R = 1$ for $t =$ "now" ⇒ $1 = \left(\frac{3}{2}\right)^{\frac{2}{3}} \left(\frac{t}{t_H}\right)^{\frac{2}{3}} \Rightarrow t_0 = \frac{2}{3} t_H$

For $\Omega_0 < 1$ (open) & $\Omega_0 > 1$ (closed), the expressions are more complicated.

(Fig. 29.6) shows present age $t_0(\Omega_0)$.

In all cases $t < t_H$ because universe is decelerating - going faster in past.

Inflation (§30.1) says universe ≈ flat, so $h = 0.71 \Rightarrow t_0 = 9.2$ Gyr, as compared to current accepted value 13.7 Gyr - not bad for simple dust model!

The Lookback Time p. 1158

$t_L = t_0 - t(z)$ = how far back in time we are looking when observing object w/ z.

Flat: $\frac{t_L}{t_H} = \frac{t_0}{t_H} - \frac{t(z)}{t_H} = \frac{2}{3} - \frac{2}{3}\frac{1}{(1+z)^{3/2}} = \frac{2}{3}\left[1 - \frac{1}{(1+z)^{3/2}}\right]$

Fig. 29.7 is t_L/t_H for a variety of Ω_0 values.

Ex. 29.1.2 p. 1159 Quasar SDSS 1030+0524 has $z = 6.28$.

Assuming flat pressureless dust universe, $\frac{t_L}{t_H} = \frac{2}{3}\left(1 - \frac{1}{7.28^{3/2}}\right) = 0.633$

$t_H = \frac{3}{2}t_0$, so $t_L/t_0 = 0.949 \Rightarrow$ only 5% of history had passed, Universe was 7.28× smaller.

Extending our Simple Model to Include Pressure p. 1160

Generalize our basic equations by interpreting ρ as mass/energy density ($m = E/c^2$).

Include pressure by using 1st Law of Thermo. $dU = dQ - dW$.

T = uniform $\Rightarrow dQ = 0$

$\frac{dU}{dt} = -\frac{dW}{dt} = -P\frac{dV}{dt} = -\frac{4}{3}\pi P\frac{d(r^3)}{dt}$

$U = \frac{4}{3}\pi r^3 u = \frac{4}{3}\pi r^3 \rho c^2 \Rightarrow \frac{d(r^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(r^3)}{dt}$

$r = R\varpi \Rightarrow \frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$ (29.50)

For $P = 0$, $R^3\rho$ = const., same as before.

This can be combined w/ (29.10) $\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2$ to give

the deceleration equation $\frac{d^2R}{dt^2} = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right)R$ (29.51)

The effect of $P > 0$ is to slow the expansion. There is no actual P force since P = uniform, but P is related to ρ via $dU = d(\rho c^2) = dW = -PdV$.

($P \ll \rho c^2$ through most of universe's history)

To solve, also need an equation of state: $P = wu = w\rho c^2$ (29.52)

Pressureless dust mass $w_m = 0$.

Blackbody radiation $P_{rad} = u_{rad}/3 \Rightarrow w_{rad} = \frac{1}{3}$.

Plug (29.52) into (29.50) $\Rightarrow R^{3(1+w)}\rho$ = constant = ρ_0 = current density (29.53)

The Deceleration Parameter

$$q(t) \equiv -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2} \quad \text{(dimensionless)}$$

In §29.3 we will see that $q(t) < 0$: the universe is accelerating.

Prob. 29.16 $\Rightarrow$ for pressureless dust universe $q(t) = \frac{1}{2}\Omega(t)$ & $q_0 = \frac{1}{2}\Omega_0 \Rightarrow$

(open, closed) $\Rightarrow q_0$ (<, >) 0.5

§29.2 The Cosmic Microwave Background

1946 George Gamow idea - cosmic abundances of elements could be produced in BB.

Published w/ Ralph Alpher (& then added Hans Bethe as 2nd author as joke - $\alpha\beta\gamma$)

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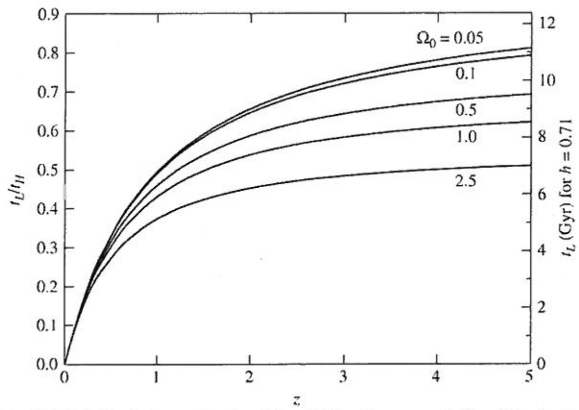

Fig. 29.7 The lookback time as a function of the redshift, z, for a range of values of the density parameter, Ω_0.

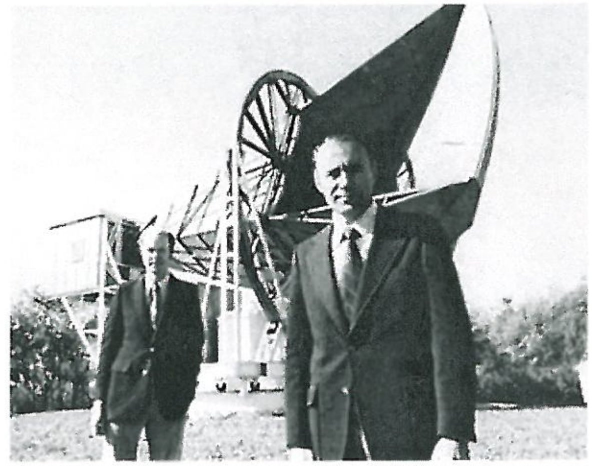

Fig. 29.8 Robert Wilson & Arno Penzias standing in front of the antenna used to first identify the cosmic microwave background.

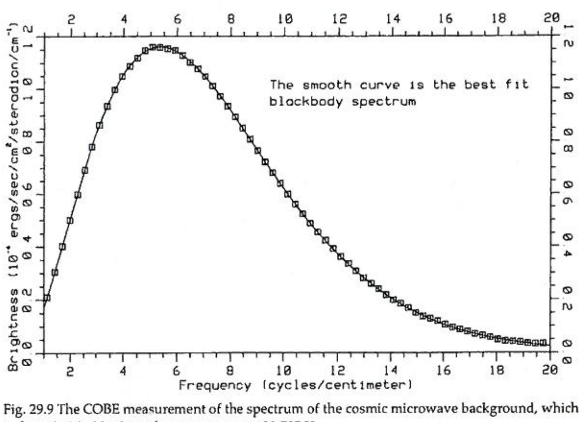

Fig. 29.9 The COBE measurement of the spectrum of the cosmic microwave background, which is that of a blackbody with a temperature of 2.725 K.

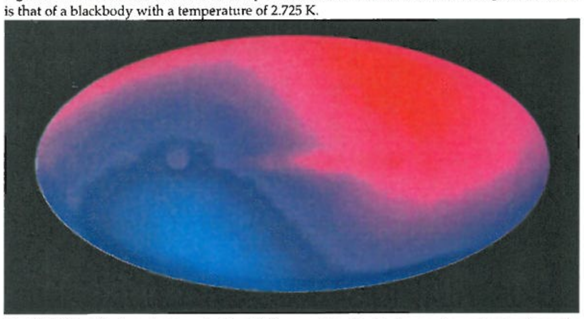

Fig. 29.10 The dipole anisotropy in the CMB caused by the Sun's peculiar velocity, shown in Galactic coordinates. Red is hotter & blue is cooler than the 2.725-K CMB.

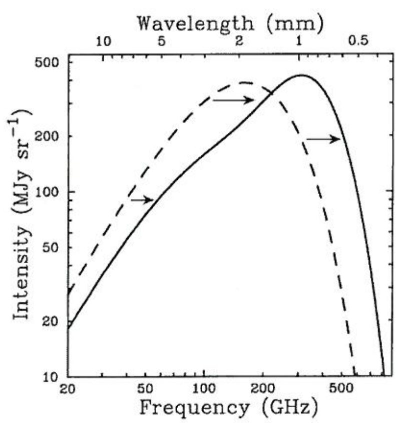

Fig. 29.11 The undistorted CMB spectrum (dashed line) & spectrum distorted by Sunyaev-Zel'dovich effect. The calculated distortion has been exaggerated by employing a fictional cluster 1000 times more massive than a typical rich cluster of galaxies.

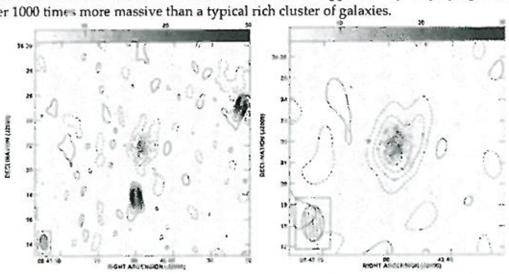

Fig. 29.12 Radio contours showing the Sunyaev-Zel'dovich effect superimposed on ROSAT images of the clusters Abell 697 (ΔT = 1047 μK, z = 0.282) & Abell 2218 (ΔT = 797 μK, z = 0.171). The dashed contours indicate a decrease in the received radio flux density.

It was later found that only H, He, & ^7_3Li could be produced in Big Bang.
At that time it was thought that H_0 was so big that t_0 was only 1-2 Gyr, shorter than age of Earth.

The Steady-State Model of the Universe p.1163

In order to avoid a Big Bang, Hermann Bondi, Thomas Gold, & Fred Hoyle at Cambridge published papers in 1948-9 proposing a steady-state universe.
As the universe expands, matter is created (out of nothing) (several H atoms/m^3/10 Gyr) to keep the density constant.
This solved the timescale problem (universe younger than Earth, stars).
Hoyle along w/ Geoffrey & Margaret Burbidge & William Fowler tried to explain cosmic abundances by stellar nucleosynthesis, but couldn't make enough He.
Although Big Bang could explain He abundance, there was no direct proof.

The Cooling of the Universe After the Big Bang p.1164

1948 Alpher & Herman paper predicted that universe is filled w/ 5K blackbody rad.
They predicted that early universe, although expanding rapidly, was filled w/ thermal radiation w/ $T \simeq 10^9$ K, baryon density $\simeq 10^{-2}$ kg m^{-3} (in order to create cosmic abundances of He).
At that time $R \simeq \left(\frac{\rho_{b,0}}{\rho_b}\right)^{1/3} = 3.47 \times 10^{-9} \Rightarrow T_0 = RT(R) \simeq 3.47$ K ($\simeq 5$K)

The Discovery of the Cosmic Microwave Background p.1165

In 1964 Robert Dicke & coworkers at Princeton calculated 10 K background radiation, apparently unaware of 16 yrs earlier Alpher & Herman calculation.
The CMB had just accidentally been found by radio astronomers Arno Penzias & Robert Wilson a few miles away at Bell Laboratories in Holmdel NJ. (Fig. 29.8)
They had been trying to get rid of a 3K hiss that came from all directions.
Penzias learned of the Dicke/Peebles calculation, called Dicke, they both published, & support for the steady-state model quickly diminished.
1991 precise measurements by COBE satellite (Fig. 29.9) $\Rightarrow T = 2.725$ K.

The Dipole Anisotropy of the CMB p.1167

There is a shift in T_{CMB} due to observer's peculiar velocity.

Prob. 29.21 $\Rightarrow$ $$T_{moving} = \frac{T_{rest}\sqrt{1-v^2/c^2}}{1-(v/c)\cos\theta}$$

θ = ∠ between direction of observation & direction of motion.
For $v \ll c$, $T_{moving} \simeq T_{rest}(1+\frac{v}{c}\cos\theta)$, 2nd term on right = dipole anisotropy (Fig. 29.10)
$\Rightarrow$ particular velocity of Sun = 370.6 km s^{-1}
Subtracting off motion of Sun wrt Milky Way, & Milky Way within Local Group $\Rightarrow$ peculiar velocity of Local Group = 627 km s^{-1} - riverlike flow towards Centaurus was discussed in §27.3.

After subtracting dipole anisotropy, CMB is incredibly isotropic, but CMB is patchwork of regions ~1° in diameter w/ $\Delta T/T_0 \sim 1/10^5$.

<u>the Sunyaev-Zel'dovich Effect p. 1169</u>

Any galaxy carried by Hubble flow observes same CMB temperature.
Evidence for this is provided by Sunyaev-Zel'dovich effect.
CMB photons passing thru hot (10^8K) intracluster gas have some fraction ($10^{-3}-10^{-2}$) scattered to higher energies by <u>inverse</u> <u>Compton</u> Scattering. (Fig. 29.11)
Effective temperature decreased by $\frac{\Delta T}{T_0} \simeq -2\frac{kT_e}{m_e c^2}\tau$ (τ = optical depth) (29.64)
Why <u>decrease</u>? Because γ's scattered out of line of sight. (???)
Typically $\Delta T/T_0 \sim$ several 10^{-4}. Fig. 29.12
Confirms cosmological nature of CMB, & is probe of rich clusters in early universe.

<u>Does CMB Constitute Preferred Frame of Reference? p. 1169</u>

No, There is no CMB "rest frame" covering entire universe.

<u>A Two-Component Model of the Universe p. 1170</u>

CMB photons have no significant gravitational effect, but did in past.
Incorporate this w/ pressure equation $P = wu$.
Matter ($v \approx 0$) has $w_m = 0$.
Relativistic particles (γ, ν) have $w_{rel} = \frac{1}{3}$. (Dark energy $w_\Lambda = -1$).
Eq. 29.53 $R^{3(1+w)}\rho = \rho_0 \Rightarrow$ different components diluted at different rates.
Energy density of CMB γ's is $u_{rad} = aT^4$, which we rewrite as $u_{rad} = \frac{1}{2}g_{rad}aT^4$.
g = degrees of freedom, depending on # of spin states & existence of antiparticles.
γ has $n_{spin} = 2$ & $n_{anti} = 1$ (it's its own antiparticle), so $g_{rad} = 2$.

<u>Neutrino Decoupling p. 1172</u>

Ignore small m_ν.
Early universe dense $\Rightarrow$ ν's have thermal spectrum, but Fermi-Dirac not Bose-Einstein.
$u_\nu = \frac{1}{2}\left(\frac{7}{8}\right)g_\nu a T_\nu^4$

g_ν = (# types) n_{anti} n_{spin} = $3 \cdot 2 \cdot 1 = 6$

3 types: e, μ, τ, each w/ antiparticle, but all ν's are left-handed (don't ask).
ν's decoupled from universe at same point, so $T_\nu \neq T_\gamma$ (not observed).

<u>The Energy Density of Relativistic Particles p. 1173</u>

It can be shown that $T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}}T$, where $T = T_{CMB}$.
For ν's + γ's we get $u_{rel} = \frac{1}{2}g_* aT^4$ where effective # of d.o.f.
$= g_* = g_{rad} + \left(\frac{7}{8}\right)g_\nu\left(\frac{4}{11}\right)^{4/3} = 3.363$. Also $\rho_{rel} = \frac{u_{rel}}{c^2}$.
This is valid back to time of e^+-e^- annihilation, $t = 1.3$s.
Density parameter $\Omega_{rel} = \frac{\rho_{rel}}{\rho_c} = \frac{4\pi G g_* a T^4}{3H^2c^2}$, $T_0 = 2.725$K $\Rightarrow \Omega_{rel,0} = 8.24 \times 10^{-5}$.

Transition from the Radiation Era to the Matter Era p1174

Since $R^4 \rho_{rel} = \rho_{rel,0}$ + $R^3 \rho_m = \rho_{m,0}$, for small R, ρ_{rel} dominated.

$\rho_{rel} = \rho_m$ when $R = R_{r,m} = \frac{\Omega_{rel,0}}{\Omega_{m,0}} = 4.16\times10^{-5}\,\Omega_{m,0}^{-1} h^{-2} = 3.05\times10^{-4}$ (WMAP)

$\Rightarrow z_{r,m} = 3270$

$RT = T_0 \Rightarrow T_{r,m} = 8920\,K$

Expansion in the Two-Component Model p1175

$$\left[\left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\left(\frac{\rho_{m,0}}{R} + \frac{\rho_{rel,0}}{R^2}\right)\right] = -kc^2 \quad (29.83)$$

$$k=0 + \text{algebra} \Rightarrow t(R) = \frac{2}{3}\frac{R_{r,m}^{3/2}}{H_0\sqrt{\Omega_{m,0}}}\left[2 + \left(\frac{R}{R_{r,m}} - 2\right)\sqrt{\frac{R}{R_{r,m}} + 1}\,\right] \quad (29.84)$$

$\Rightarrow t_{r,m} = 5.52\times10^4\,yr$

radiation era ($t < t_{r,m}$) $R \propto t^{\frac{1}{2}}$, $T \propto t^{-\frac{1}{2}}$

matter era ($t > t_{r,m}$) $R \propto t^{2/3}$

$$\frac{t(z)}{t_H} = \frac{2}{3}\frac{1}{(1+z^2)^{3/2}\sqrt{\Omega_{m,0}}} \quad (29.92)$$

Now $= t(z=0) = 12.5$ Gyr (WMAP) (still ~1 Gyr too short).

Big Bang Nucleosynthesis p.1177

At $t \sim 10^{-4}$ s, $T \sim 10^{12}$ K, universe a mixture of $\gamma, e^{\pm}, \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu$.

Also about 5 n's + p's per 10^{10} photons, constantly being converted to one another.

$\rightarrow (m_n - m_p)c^2 = 1.293$ MeV (misprint in book p.1177 — they have $m_p - m_n$)

At $T = 10^{12}$ K, $kT = 86$ MeV, so $\frac{n_n}{n_p} = e^{-(m_n - m_p)c^2/kT} = 0.985$ (another misprint).

fast [As T↓, n_n/n_p↓ until $T = 10^{10}$ K, $n_n/n_p = 0.223$, at which point #'s of $e^{\pm}$ dropped so that n's were no longer being created.

then n's decayed to p's w/ $T_{\frac{1}{2}} = 614$ s for 176 s while $T = 10^{10} \rightarrow 10^9$ K.

At this point it is cool enough for nucleosynthesis (Fig. 29.13)

This agrees well w/ observed abundances (Fig. 29.14) (explain all)

The Origin of the Cosmic Microwave Background

In the early universe, e's + γ's were in equilibrium.

When $t \sim 10^6$ yr, recombination occurred (formation of neutral atoms, no more free e's, universe became transparent).

The photons of the CMB were last scattered then.

The Surface of Last Scattering p1181

... is a spherical surface, centered on Earth, from which CMB photons last scattered before traveling to Earth.

67b

4/12/16

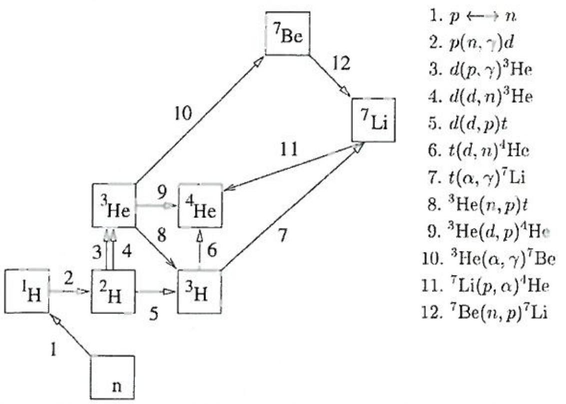

Fig. 29.13 The reaction network that is responsible for Big Bang nucleosynthesis. "d" stands for deuterium & "t" is tritium.

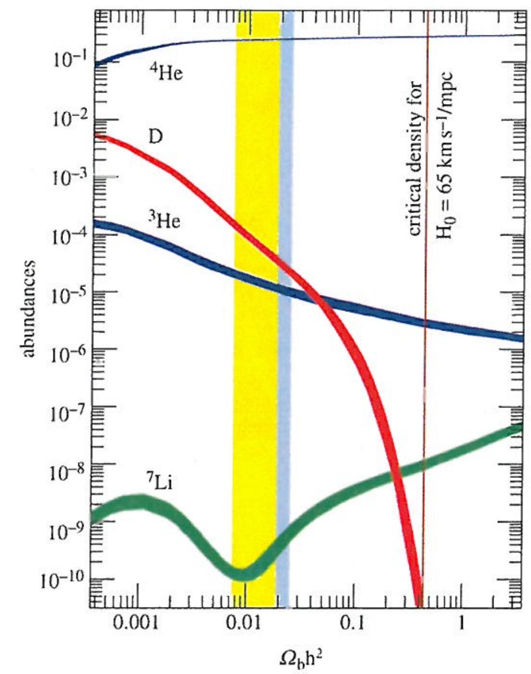

Fig. 29.14 Calculated mass abundances as a function of present density of baryonic matter in the universe. The yellow bar delineates the consistence interval, the range of $\Omega_{b,0}h^2$ that agree with observed abundances. The blue stripe corresponds to abundances of primeval deuterium measured for molecular clouds in front of quasars. WMAP value of 0.0224 runs down center of blue stripe, & WMAP value of critical density 0.504 is shown at right.

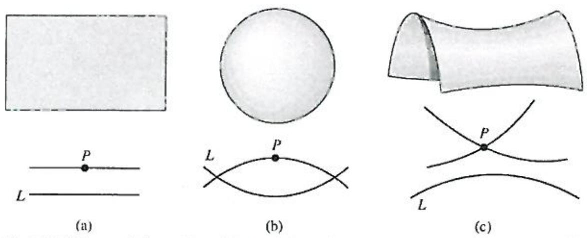

Fig. 29.15 The parallel postulate, illustrated for 3 alternative geometries: (a) Euclidean, (b) elliptic, & (c) hyperbolic.

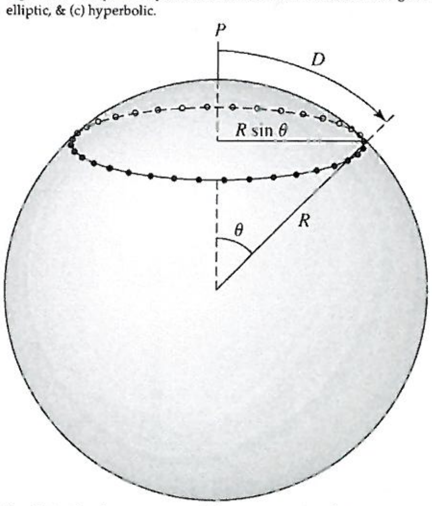

Fig. 29.16 A local measurement of the curvature of a sphere.

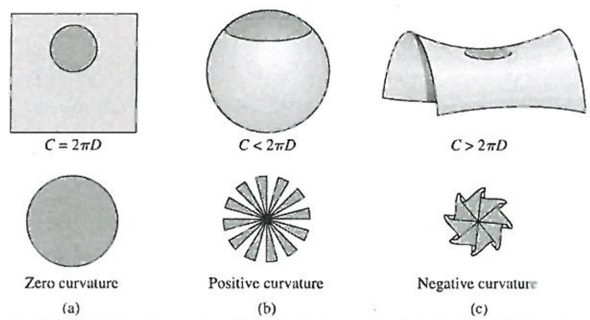

Fig. 29.17 Calculating the curvature of a surface in 3 geometries: (a) a flat plane, (b) the surface of a sphere, & (c) the surface of a hyperboloid.

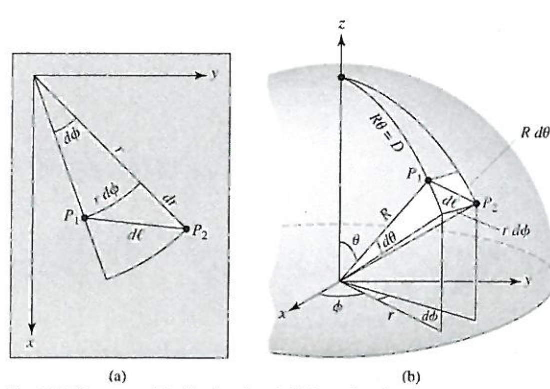

Fig. 29.18 dl is measured for (a) a flat plane & (b) the surface of a sphere.

Everything before this z (recombination, decoupling) is hidden from view. It actually has a thickness Δz.

The Conditions at Recombination p.1181

the temperature at recombination can be estimated by the Saha Eq. (Ch.8) (skip)

Using our model gives reasonable estimates for z + T at decoupling.

More precise values from WMAP: $[z_{dec}]_{WMAP} = 1089 \pm 1$

$T_{dec} = T_0 (1+z_{dec}) = 2970\,K$, also $[t_{dec}]_{WMAP} = 379^{+8}_{-7}$ kyr,

$[\Delta t_{dec}]_{WMAP} = 118^{+3}_{-2}$ kyr, $[\Delta z_{dec}]_{WMAP} = 195 \pm 2$

So decoupling (recombination) occured when the univers was 200-500 kyr old.

The Dawn of Precision Cosmology p.1183

A couple of nice quotes from Steven Weinberg + John Bahcall about how amazing it is that our science can actually describe the early universe.

§29.3 Relativistic Cosmology

Now we need to get some understanding of curved spacetime.

Euclidean, Elliptic + Hyperbolic Geometries

Foundations of plane (flat) geometry laid by Euclid ~300 B.C.

Euclid's "Elements" contains 465 theorems derived from 5 postulates, or self-evident truths.

1. It is possible to draw a straight line from any point to any point.
2. It is possible to extend a finite straight line continuously in a straight line.
3. It is possible to describe a circle with any center + radius.
4. All right angles are equal.

the 5th is more complicated.

5. If a (straight) line falling on 2 lines makes the interior angles on one side less than 2 right angles, the 2 lines, if extended indefinitely, meet on the side on which the angles are less than 2 right angles.

It is equivalent to the statement that given a line L + point P, there is one + only one line through P parallel to L (parallel lines never meet).

Mathematicians thought the 5th might be derivable from the other 4, so in the 1700's they tried proof by contradiction - assuming it false. But instead they found they were developing alternative (non-Euclidean) geometries.

there are 3 versions of Postulate 5 which have been proved (in 1868) to be as logically consistent as Euclid's.

5b (Riemann) Given line L + point P in a plane, there exists <u>no</u> line thru P parallel to L. (Fig. 29.15b) This describes <u>elliptic</u> geometry.

5c - 5^{th} postulate of hyperbolic geometry (Gauss, Bolyai, Lobachevski): There exist <u>at least 2</u> lines parallel to L. (Fig. 29.15c)

Elliptic: $\Sigma \angle$'s triangle $> 180°$, $C < 2\pi R$.

Hyperbolic: $\Sigma \angle$'s triangle $< 180°$, $C > 2\pi R$.

Geometry of universe must be determined empirically.

Gauss (1820) used 3 mountains, longest side = 107 km, found $\Sigma \angle = 180°$ (to within experimental error) - not sensitive enough.

<u>The Robertson-Walker Metric for Curved Spacetime</u> p1185

Schwarzschild metric valid only <u>outside</u> matter.

Instead use cosmological principle - curvature same everywhere at same t.

It is possible to <u>locally</u> measure curvature K.

Carroll & Ostlie consider an ant on a sphere (Fig. 29.16)

Make series of dots distance $D = R\theta$ from point P, measure circumference C_{meas}.

Expected $C_{exp} = \pi D = \pi R\theta$

$C_{meas} = 2\pi R \sin\theta = 2\pi R \sin(D/R)$ $\rightarrow \lim D \to 0$

Curvature K turns out to be $6\pi \times$ (fractional discrepancy in C) / A_{exp}

$$= 6\pi \frac{C_{exp} - C_{meas}}{C_{exp} A_{exp}} = 6\pi \frac{2\pi D - 2\pi R \sin(D/R)}{(2\pi D)(\pi D^2)} = \frac{6}{D^3}\left\{D - R\left[\frac{D}{R} - \frac{1}{3!}\left(\frac{D}{R}\right)^3 + \frac{1}{5!}\left(\frac{D}{R}\right)^5 - \dots\right]\right\}$$

$$= \frac{1}{R^2} - \frac{1}{20}\frac{D^2}{R^4} + \dots$$

$$K = \lim_{D\to 0}\left(\frac{1}{R^2} - \frac{1}{20}\frac{D^2}{R^4} + \dots\right) = \frac{1}{R^2}$$

In general, $K = \lim_{D \to 0} \frac{6\pi (C_{exp} - C_{meas})}{(2\pi D)(\pi D^2)} = \frac{3}{\pi} \lim_{D\to 0} \frac{2\pi D - C_{meas}}{D^3}$

(Fig. 29.17) demonstrates zero, positive, + negative curvature.

Flat space distance between points P_1 & P_2 $(d\ell)^2 = (dr)^2 + (r d\phi)^2$ (Fig. 29.18a)

On surface of sphere (29.18b) $(d\ell)^2 = (R d\theta)^2 + (r d\phi)^2$, which can easily be shown to be (in text) $(d\ell)^2 = \left(\frac{dr}{\sqrt{1 - r^2/R^2}}\right)^2 + (r d\phi)^2$

$K = \frac{1}{R^2}$ for sphere, so in general $(d\ell)^2 = \left(\frac{dr}{\sqrt{1 - Kr^2}}\right)^2 + (r d\phi)^2$

Adding in 3^{rd} spatial dimension θ + t gives

$$(ds)^2 = (c\,dt)^2 - \left(\frac{dr}{\sqrt{1-Kr^2}}\right)^2 - (r\,d\theta)^2 - (r \sin\theta\, d\phi)^2$$

Now use $r(t) = R(t)\varpi$ & define time-independent curvature k by $K(t) = \frac{k}{R^2(t)}$ (29.105) $\Rightarrow$

$$\boxed{(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]} \quad (29.106)$$

Robertson-Walker metric - showed independently in mid-1930's that it's the most general metric for isotropic, homogeneous universe.
ϖ defined by requirement that now ($R_0 = 1$) area of spherical surface centered at $\varpi = 0$ is $4\pi\varpi^2$, ϖ is comoving coordinate.

<u>The Friedmann Equation p.1190</u>

Solving Einstein field eq. leads to F.eq., which is same as Newtonian eq. 29.10.

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2$$

Named after Russian meteorologist & mathematician Aleksandr Friedmann (1888-1925).

<u>The Cosmological Constant</u>

Before Hubble expansion discovered, Einstein added cosmological constant Λ to allow static universe. It changes Friedmann eq. to

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2 \quad (29.108)$$

this has the effect of adding an outwards force $\vec{F}_\Lambda = \frac{1}{3}\Lambda m c^2 r\hat{r}$ (for shell of mass m).

When expansion was discovered, Einstein called Λ "the biggest blunder of my life", but now it has reappeared as..

<u>The Effects of Dark Energy</u>

$\Lambda = $ const. $\Rightarrow$ $\rho_\Lambda \equiv \frac{\Lambda c^2}{8\pi G} = $ const. $= \rho_{\Lambda,0}$ $\Rightarrow$ deceleration eq. becomes

$$\frac{d^2R}{dt^2} = \left\{-\frac{4}{3}\pi G\left[\rho_m + \rho_{rel} + \rho_\Lambda + \frac{3(P_m + P_{rel} + P_\Lambda)}{c^2}\right]\right\}R \quad (29.116)$$

w/ dark energy pressure $P_\Lambda = -\rho_\Lambda c^2$ ($w_\Lambda = -1$)

So Λ has $\rho_\Lambda > 0$ but $P_\Lambda < 0$.

Friedmann eq. can also be written $H^2\left[1 - (\Omega_m + \Omega_{rel} + \Omega_\Lambda)\right]R^2 = -kc^2$ (29.117)
where k is now interpreted as present value of curvature K (from 29.105)

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\Lambda c^2}{3H^2}$$

With $\Omega = \Omega_m + \Omega_{rel} + \Omega_\Lambda$, Friedmann eq. becomes

$$H^2(1-\Omega)R^2 = -kc^2 \quad (29.120) \Rightarrow H_0^2(1-\Omega_0) = -kc^2 \quad (29.121)$$

Putting in the z-dependencies of the Ω's gives

$$H = H_0(1+z)\left[\Omega_{m,0}(1+z) + \Omega_{rel,0}(1+z)^2 + \frac{\Omega_{\Lambda,0}}{(1+z)^2} + 1 - \Omega_0\right]^{\frac{1}{2}} \quad (29.122)$$

WMAP values: $\Omega_{m,0} = 0.27 \pm 0.04$, $\Omega_{rel,0} = 8.24\times10^{-5}$, $\Omega_{\Lambda,0} = 0.73 \pm 0.04$

$\Omega_0 = \Omega_{m,0} + \Omega_{rel,0} + \Omega_{\Lambda,0} = 1$ $\quad [\Omega_0]_{WMAP} = 1.02 \pm 0.02$

Prob. 29.47 $\Rightarrow$ deceleration parameter $q(t) = \frac{1}{2}\sum_i (1+3w_i)\Omega_i(t) = \frac{1}{2}\Omega_m + \Omega_{rel} - \Omega_\Lambda$

$q_0 = -0.60$ (WMAP) (accelerating)

√ Λ decouples geometry from dynamics: we may have flat ($k=0$) universe w/ acceleration, a combination not possible w/ dust universe.

The Λ Era p. 1194

$\rho_\Lambda = $ const., $\rho_m \propto R^{-3}$, + $\rho_{rel} \propto R^{-4}$. the mass era has no segued into the Λ era.

Transition occurred when $\rho_m = \rho_\Lambda \Rightarrow R_{m,\Lambda} = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{\frac{1}{3}} = 0.72$

$z_{m,\Lambda} = \frac{1}{R_{m,\Lambda}} - 1 = 0.39$

The universe changed from deceleration to acceleration at $R_{accel} = 2^{-\frac{1}{3}} R_{m,\Lambda} = 0.57$, $z_{accel} = 0.76$

Λ was negligible in early universe, so all our early universe results of §29.1-2 are still valid.

Must numerically integrate to find $t(R)$ (Fig. 29.19)

Best estimate $[t_0]_{WMAP} = 13.7 \pm 0.2$ Gyr

$t_{accel} = 7.08$ Gyr, so universe has been accelerating for the 2nd half of its existence.

Also $t_0 = 0.993 t_H$ (deceleration + acceleration nearly cancel)

To good accuracy can neglect radiation era ($t_{r,m} \simeq 5\times10^4$ yr) to give

$$R(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2} H_0 t \sqrt{\Omega_{\Lambda,0}}\right) \quad \text{(Fig. 29.20)}$$

For $t \gg t_H$ this is $R(t) \simeq \left(\frac{\Omega_{m,0}}{4\Omega_{\Lambda,0}}\right)^{\frac{1}{3}} e^{H_0 t\sqrt{\Omega_{\Lambda,0}}}$ - exponential expansion!

Model Universes on the $\Omega_{m,0} - \Omega_{\Lambda,0}$ Plane p. 1197

Fig. 29.21 Models specified by $\Omega_{rel,0}$, $\Omega_{m,0}$, + $\Omega_{\Lambda,0}$, but $\Omega_{rel,0}$ negligible.

$k=0 \Rightarrow \Omega_{m,0} + \Omega_{\Lambda,0} = 1$ line divides closed + open universes.

(29.123) $q(t) = \frac{1}{2}\sum_i (1+3w_i)\Omega_i(t) = \frac{1}{2}\Omega_m(t) - \Omega_\Lambda(t) = 0 \Rightarrow$

$\Omega_{m,0} - 2\Omega_{\Lambda,0} = 0$ line divides decelerating + accelerating.

The line between "expands forever" + "Recollapses" depends on whether or not there are positive real solutions of $dR/dt = 0$.

t_0 increases towards the upper left, + becomes infinite in the "No Big Bang" region.

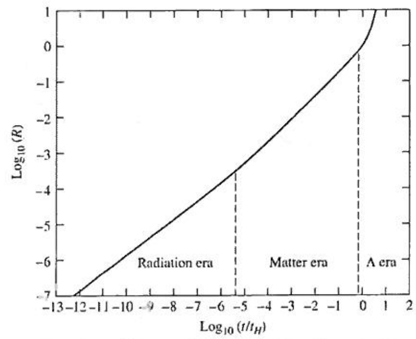

Fig. 29.19 A logarithmic graph of the scale factor R as a function of time. $R \propto t^{1/2}$ during the radiation era, $t^{2/3}$ during the matter era, & R grows exponentially during the Λ era.

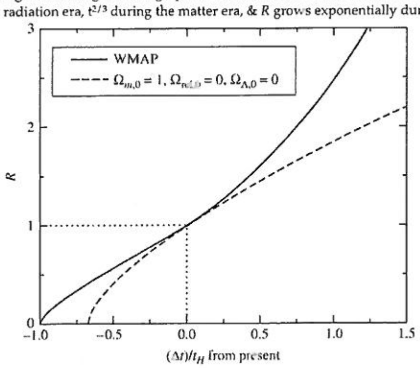

Fig. 29.20 Scale factor R for WMAP universe with $t_0 \simeq t_H$, & a flat one-component universe of pressureless dust with $t_0 = 2t_H/3$.

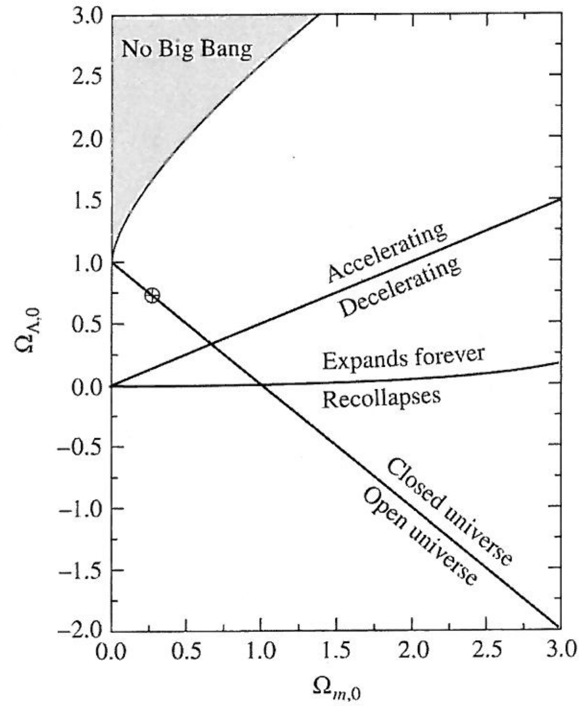

Fig. 29.21 Every point on this plane represents a possible universe. The point $(\Omega_{m,0} = 0.27, \Omega_{\Lambda,0} = 0.73)$ is indicated by the circle.

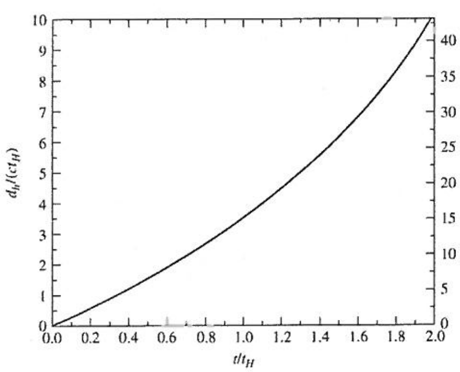

Fig. 29.22 The proper distance from Earth to the particle horizon, using WMAP values.

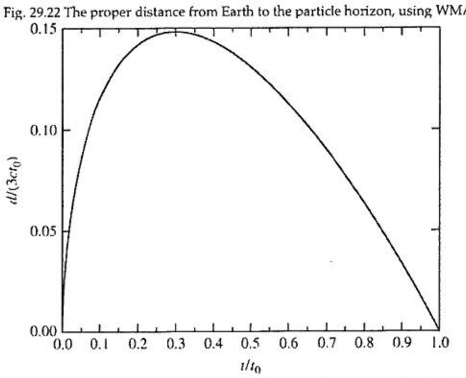

Fig. 29.23 The proper distance from Earth of a photon emitted from the present particle horizon at the time of the Big Band.

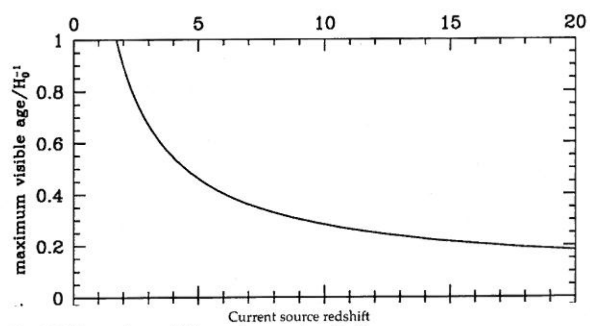

Fig. 29.24 The maximum visible age of a source in units of t_H.

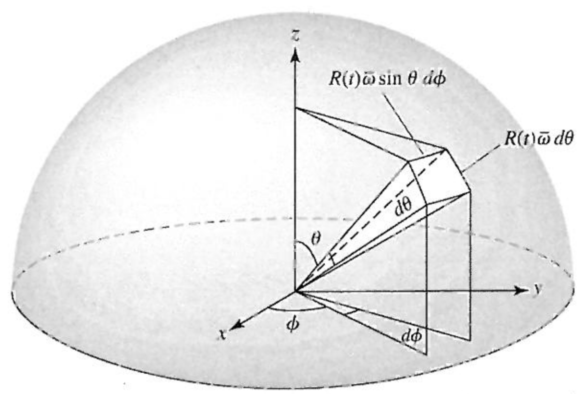

Fig. 29.25 An element of area on the surface of a sphere centered at $\bar{\omega} = 0$. Integrating over the angles θ & ϕ shows that the surface area of the sphere is $4\pi\,[R(t)\bar{\omega}]^2$.

In that region, there is an unstable equilibrium between the inwards pull of gravity & the outward push of dark energy.
These represent "bouncé" universes, which have a maximum redshift at bounce of $z_{bounce} \le 2\cos\left[\frac{1}{3}\cos^{-1}\left(\frac{1-\Omega_{m,0}}{\Omega_{m,0}}\right)\right]-1$

Since we observe larger-z objects, our universe is not a bounce universe, unless bounce occurred at earlier era (< radiation)
We can determine our Universe by observing q_0 (§29.4) & Ω_0 (Ch. 30).

§29.4 Observational Cosmology p. 1199

The Origin of the Cosmological Redshift

Use Robertson-Walker metric (29.106) w/ $ds=0$ (photon) & $d\theta=d\phi=0$.
Photon emitted at ϖ_e & received at Earth at $\varpi=0$. Photon moves inward ⇒

$$-\frac{c\,dt}{R(t)}=\frac{d\varpi}{\sqrt{1-k\varpi^2}}$$

If a wave crest is emitted at ϖ_e at t_e & observed at Earth ($\varpi=0$) at t_0,

$$\int_{t_e}^{t_0}\frac{c\,dt}{R(t)}=-\int_{\varpi_e}^{0}\frac{d\varpi}{\sqrt{1-k\varpi^2}}=\int_0^{\varpi_e}\frac{d\varpi}{\sqrt{1-k\varpi^2}}$$

The next wavefront is emitted at $t_e+\Delta t_e$ & observed at $t_0+\Delta t_0$.

$$\int_{t_e+\Delta t_e}^{t_0+\Delta t_0}\frac{c\,dt}{R(t)}=\int_0^{\varpi_0}\frac{d\varpi}{\sqrt{1-k\varpi^2}}=\int_{t_e}^{t_0}\frac{c\,dt}{R(t)}$$

$$\cancel{\int_{t_e+\Delta t_e}^{t_0}}+\int_{t_0}^{t_0+\Delta t_0}=\int_{t_e}^{t_e+\Delta t_e}+\cancel{\int_{t_e+\Delta t_e}^{t_0}}$$

w/ $R(t_e+\Delta t_e)\simeq R(t_e)$ & $R(t_0+\Delta t_0)\simeq R(t_0)=1 \Rightarrow \Delta t_0=\frac{\Delta t_e}{R(t_e)}$

Then $\lambda=c\Delta t \Rightarrow R(t_e)=\frac{\lambda_0}{\lambda_e}=1+z \Rightarrow$ it is due to expansion of space.
This also gives a cosmological time dilation $\Delta t_0/\Delta t_e=1+z$, which seems to be consistent w/ temporal changes in a Type I SN at $z=0.361$. (universe is a big ball of Dr. Who: "... wibbly wobbley timey wimey..." stuff.

Distances to the Most Remote Objects in the Universe p. 1201

Proper distance of object from Earth $d_p=\sqrt{-(ds)^2}$ w/ $dt=d\theta=d\phi=0$ ⇒
Robertson-Walker (29.106) $d_p(t)=R(t)\int_0^{\varpi}\frac{d\varpi'}{\sqrt{1-k\varpi'^2}}=R(t)\,d_{p,0}$

Integrating gives: $k=0$ (flat) ⇒ $d_{p,0}=\varpi$, proper distance = coordinate distance

$k>0$ (closed), $d_{p,0}=\frac{1}{\sqrt{k}}\sin^{-1}(\varpi\sqrt{k})>\varpi$

$k<0$ (open), $d_{p,0}=\frac{1}{\sqrt{|k|}}\sinh^{-1}(\varpi\sqrt{|k|})<\varpi$

Remember, ϖ is defined by area $=4\pi\varpi^2$

For $k>0$, solving for ϖ gives $\varpi=\frac{1}{\sqrt{k}}\sin(d_{p,0}\sqrt{k})$

If $d_{p,0}$ is the proper distance to point X, then X is also at radial distances $d_{p,0} + 2\pi n/\sqrt{k}$, n = integer. This is like an ant on the N pole circling the Earth n times before stopping at the final destination. Universe is finite unbounded.
But you couldn't really make that trip. A photon circumnavigating the universe would return just in time for the Big Crunch.
Define circumference of universe $C_{univ}(t) = \frac{2\pi R(t)}{\sqrt{k}}$
The radius of curvature is $R(t)/\sqrt{k}$.

The Particle Horizon & the Horizon Distance p.1203

As $t\uparrow$, photons from increasingly distant objects have time to get here.
Proper distance to this particle horizon is the horizon distance $d_h(t)$.
Observer at $\varpi = 0$, particle horizon at ϖ_e at time $t \Rightarrow$ photon emitted at ϖ_e at $t=0$ reaches $\varpi = 0$ at t. Analogy: turn on lights everywhere at t=0.
$(29.145) \Rightarrow d_h(t) = R(t)\int_0^t \frac{c\,dt'}{R(t')}$
Radiation era $R(t) = Ct^{\frac{1}{2}}$, C = const. $\Rightarrow d_h(t) = Ct^{\frac{1}{2}}\int_0^t \frac{c\,dt'}{Ct'^{\frac{1}{2}}} = 2ct$.
Now, in Λ era, $d_h(t)$ = numerical solution (Fig. 29.22)
Now, $t_0 = 0.993\,t_H = 13.7$ Gyr $\Rightarrow d_{h,0} = 14.6$ Gpc (= 47.6 Gly > 13.7 Gly)
It can be shown that the proper distance today to the farthest object that will ever be observable in the future is 19.3 Gpc.
In the future, particle horizon & scale factor both grow exponentially.
Eventually, particle at horizon will remain at horizon as universe expands, with photons becoming redder & fewer.

Ex. 29.4.1 p. 1205 Show that at time He nuclei formed, t = 178 s, d_h = 0.7 AU
That region has now expanded to 1.3 kpc, but the causally connected region is vastly larger.

The Arrival of Photons p. 1206

Since universe much smaller when CMB photons emitted, why did it take so long (age of universe) to get here?
Path of inward-moving photon from R-W metric w/ $ds = d\theta = d\phi = 0 \Rightarrow c\,dt = -R\frac{d\varpi}{\sqrt{1-k\varpi^2}}$ (29.106) p.70
Flat, matter-dominated universe $\Rightarrow k = 0$ & $R(t) = (t/t_0)^{2/3} \Rightarrow$
$$\int_{t_e}^{t}\frac{c\,dt'}{R(t')} = 3ct_0\left[\left(\frac{t}{t_0}\right)^{\frac{1}{3}} - \left(\frac{t_e}{t_0}\right)^{\frac{1}{3}}\right] = -\int_{\varpi_e}^{\varpi} d\varpi = \varpi_e - \varpi$$
Photon gets here ($\varpi = 0$) now ($t = t_0$) $\Rightarrow \varpi_e = 3ct_0\left[1 - \left(\frac{t_e}{t_0}\right)^{\frac{1}{3}}\right] \Rightarrow$
$$\varpi = 3ct_0\left[1 - \left(\frac{t}{t_0}\right)^{\frac{1}{3}}\right]$$
Proper distance to photon: (29.106) w/ $dt = d\theta = d\phi = 0$:
$$d_p(t) = \int\sqrt{-(ds)^2} = R(t)\int_0^{\varpi} d\varpi = R(t)\varpi = 3ct_0\left[\left(\frac{t}{t_0}\right)^{\frac{2}{3}} - \frac{t}{t_0}\right]$$ (Fig. 29.23)
Photon gets farther away before getting closer.

If photon emitted at $t_e = t_{dec} = 379\,kyr$, at that time it was at

$$d_p(t_e) = 3ct_0\left[\left(\frac{t_{dec}}{t_0}\right)^{\frac{2}{3}} - \frac{t_{dec}}{t_0}\right] \simeq 3ct_0\left(\frac{t_{dec}}{t_0}\right)^{\frac{2}{3}} = 3c\left(t_0 t_{dec}^2\right)^{\frac{1}{3}}$$

$$= 3c\left(13.7\times10^9 yr\,(379\times10^3 yr)^2\right)^{\frac{1}{3}} = 3.76\times10^7 ly$$ (not in text).

The Maximum Visible Age of a Source p. 1208

Are there presently visible objects that will become invisible in future due to expansion?

Light emitted at t_e observed now (t_0), then light emitted by same ϖ at future time t_i, reaching Earth at t_f: $\int_{t_e}^{t_0} \frac{dt}{R(t)} = \int_{t_i}^{t_f} \frac{dt}{R(t)}$

The maximum visible age (mva) of an object observed at z is the t_i value for which $t_f = \infty$.

As time goes on, the galaxy will become frozen in time + the photons will become more infrequent + red-shifted.

(Fig. 29.24) $z > 1.8 \Rightarrow t_{mva} < t_H \Rightarrow$ we will never see it as it is today.

$z = 5\text{-}10 \Rightarrow$ objects will be seen only as they were when universe 4-6 Gyr old.

As universe ages sky becomes increasingly empty + universe becomes causally fragmented (but not four galaxy, which is gravitationally bound).

the Comoving Coordinate $\varpi(z)$ p 1209

Use $d_{p,0} = \int_{R(t_e)}^{R(t_0)} \frac{c\,dt}{R(t)} = \frac{c}{H_0} H_0 \int_{1/1+z}^{1} \frac{dR}{R\,(dR/dt)} = \frac{c}{H_0} I(z)$

$$I(z) = H_0 \int_{\frac{1}{1+z}}^{1} \frac{dR}{R\,(dR/dt)} = H_0 \int_0^z \frac{dz'}{H(z')}$$

Find $\varpi(z) = \frac{c}{H_0} S(z)$ where $S(z) = \begin{cases} I(z) & \Omega_0 = 1 \\ \frac{1}{\sqrt{\Omega_0 - 1}} \sin\left[I(z)\sqrt{\Omega_0 - 1}\right] & \Omega_0 > 1 \\ \frac{1}{\sqrt{1-\Omega_0}} \sinh\left[I(z)\sqrt{1-\Omega_0}\right] & \Omega_0 < 1 \end{cases}$

For $z \ll 1$ find $\varpi \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right]$

The Proper Distance p 1211

For $z \ll 1$, $d_{p,0} \simeq \varpi(z) \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right]$

So the simple Hubble law $d = \frac{cz}{H_0}$ is accurate to 10% for $z < 0.13$

the Luminosity Distance

If F = bolometric flux + L = luminosity, luminosity distance d_L defined

by $d_L^2 = \frac{L}{4\pi F}$, (Fig. 29.25 - skip)
They show that $d_L(z) = \varpi(1+z) \simeq \frac{cz}{H_0}\left[1 + \frac{1}{2}(1-q_0)z\right]$ $(z \ll 1)$

The Redshift-Magnitude Relation p. 1212

Plugging d_L into the formula for the distance modulus gives the redshift-magnitude relation, which allows us to measure q_0.

$$m - M = 5\log_{10}(d_L/10\text{pc}) \simeq 42.38 - 5\log_{10}(h) + 5\log_{10}(z) + 1.086(1-q_0)z \quad (z \ll 1)$$

(Fig. 29.26)

In mid-1990's, 2 groups observed SN Ia at cosmological distances, found they were ~0.25 mag dimmer at $z \simeq 0.5$ than should be for $\Omega_\Lambda = 0$.
After eliminating other effects ("gray dust", evolutionary effects...)
$\Rightarrow \Omega_{\Lambda,0} = 0.7$, acceleration (Fig. 29.27, 29.28)
For $z > z_{accel} = 0.76$, should see deceleration (SNe "too bright")
(Fig. 29.29) these observations rule out "gray dust" + evolution.

Angular Diameter Distance p. 1215

Angular diameter distance $d_A \equiv \frac{D}{\theta}$, D = linear diameter, θ = angular diameter.
Proper distance across galaxy from R-W metric w/ $dt = d\varpi = d\phi = 0$

$$(ds)^2 = -R^2\varpi^2(d\theta)^2 = -D^2 \Rightarrow D = R(t_e)\varpi\theta = \frac{\varpi\theta}{1+z} = \frac{c}{H_0}\frac{S(z)\theta}{1+z}$$

$$\rightarrow d_A = \frac{c}{H_0}\frac{S(z)}{1+z}$$

(Fig. 29.30) shows $\frac{c\theta}{H_0 D} = \frac{1+z}{S(z)}$: Beyond a certain z, θ increases w/ D.
But galaxies do not have sharp boundaries, + they evolve.
Still, useful results have come from using the Sunyaev-Zel'dovich effect (inverse Compton scattering of CMB photons) + modeling hot intracluster gas.

Ch. 30 The Early Universe p. 1230

§30.1 The Very Early Universe + Inflation

Fundamental Particles (Table 30.1)

Standard Model $\Rightarrow$ 3 kinds of fundamental particles.
3 charged leptons e, μ, τ, associated neutrinos ν_e, ν_μ, ν_τ + antiparticles, Fermions.
6 quarks: up, down, strange, charm, bottom, top, each of 3 "colors" - fermions
13 force-carrying particles - photon, 8 gluons, 3 vector gauge bosons ($W^\pm, Z^0$) (weak interaction), scalar Higgs bosons (provides mass) - all bosons.
Particle accelerators can reproduce conditions back to quark-hadron transition, 10^{-5} s.
(Table 30.2) shows early times + temperatures.

(Is Higgs field a 5th force? Don't know, it's complicated)

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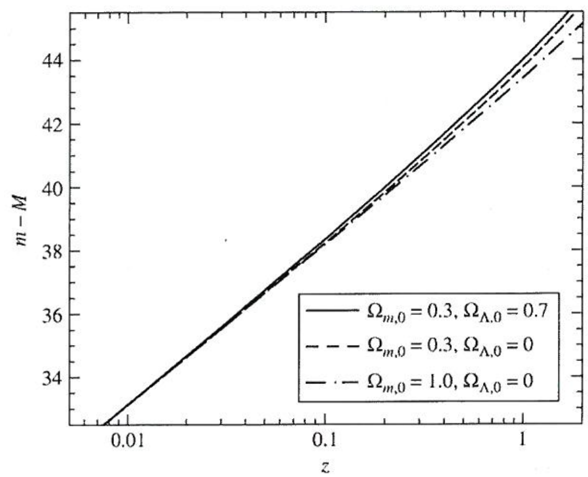

Fig. 29.26 The redshift-magnitude relation for $h = 0.71$ & several values of $\Omega_{m,0}$ & $\Omega_{\Lambda,0}$.

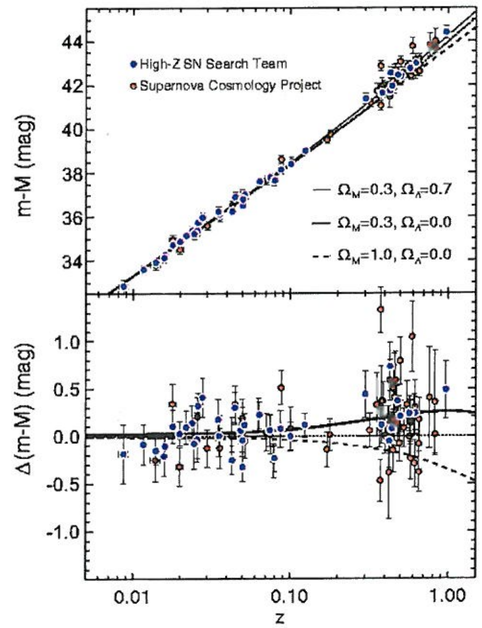

Fig. 29.27 The redshift-magnitude relation measured for high-z supernovae. The K-correction has been applied to the apparent magnitudes. The lower graph shows the data after subtracting the theoretical curve for $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0$.

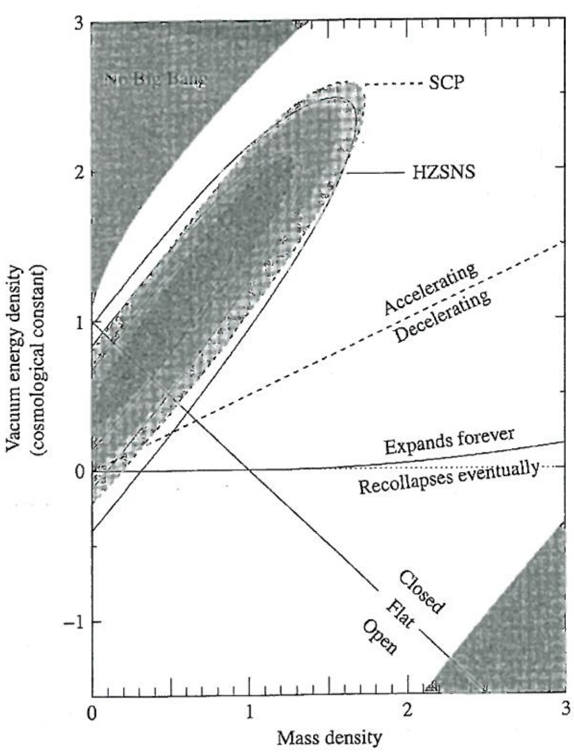

Fig. 29.28 The location of the most probably values of $\Omega_{m,0}$ & $\Omega_{\Lambda,0}$ for high-z supernovae.

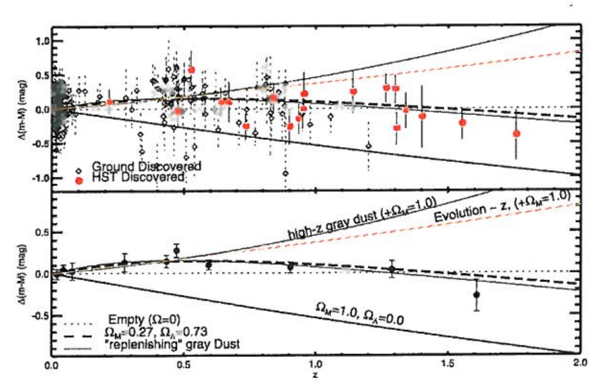

Fig. 29.29 The redshift-magnitude relation measured for very high-z supernovae. The K-correction has been applied, & the theoretical curve for $\Omega = 0$ (a "coasting universe") has been subtracted. The lower graph illustrates the averages of binned data (grouped according to redshift) & compares them to curves of alternative models incorporating "gray dust" or evolutionary effects for supernovae.

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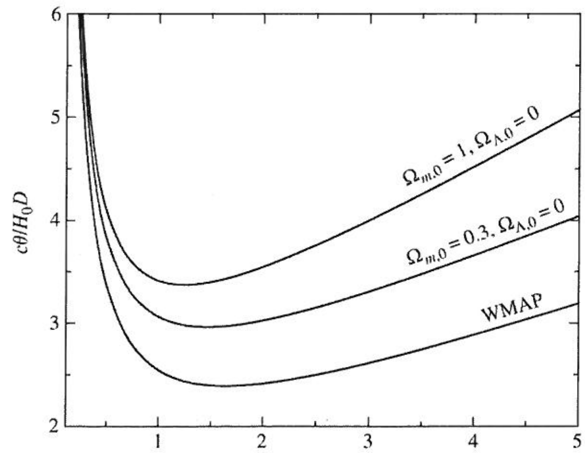

Fig. 29.30 The angular diameter θ of a galaxy in units of H_0D/c for several values of $\Omega_{m,0}$ & $\Omega_{\Lambda,0}$.

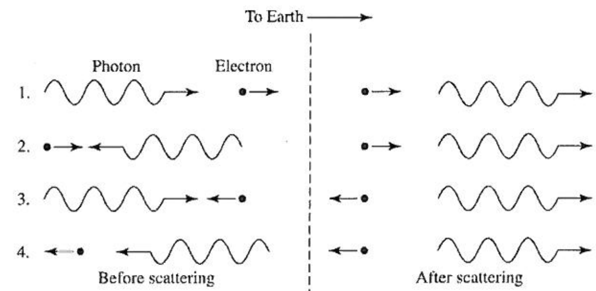

Fig. 29.31 Inverse Compton scattering of a CMB photon by a high-energy electron.