Simpson's Rule

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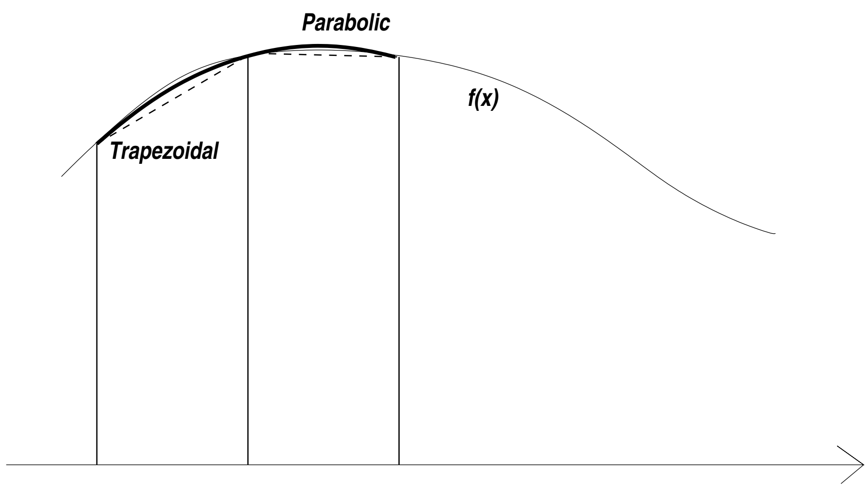
The midpoint rule involves the construction of a set of rectangles to estimate the area under a curve. The trapezoidal rule involves the construction of trapezoids to estimate the area under a curve. These are both linear methods for estimating area. It is natural to ask, can we use a parabola (quadratic methods) to give an estimate of the area i.e. Simpson's rule.

The purpose of using a quadratic approximation is to take advantage of estimating the integral using as few points as necessary in the case that the function has relatively large concavity .



An example of the trapezoidal and parabolic estimates.

We need to find the area under a parabola given by the three points

$$(0, f(0));\ (1, f(1));\ (2, f(2))$$

The parabola is

$$p(x) = f(0)\frac{(x-1)(x-2)}{2} - f(1)x(x-2) + f(2)\frac{x(x-1)}{2}$$

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or

$$p(x) = \frac{1}{2}\Big((f(0) - 2f(1) + f(2))x^2 + (-3f(0) + 4f(1) - f(2))x + 2f(0)\Big)$$

The integral gives

$$\int_0^2 p(x)\ dx = \frac{1}{3}\Big(f(0) + 4f(1) + f(2)\Big)$$

If we rescale taking

$$(0, f(0));\ \ (h, f(h));\ \ (2h, f(2h))$$

the the integral gives

$$\int_0^{2h} p(x)\ dx = \frac{h}{3}\Big(f(0) + 4f(h) + f(2h)\Big).$$

Patching together several parabolic arcs gives Simpson's Rule

$$\int_a^b f(x)\ dx \approx \frac{h}{3}\sum_{i=1}^{n}\Big(f(x_o) + 4(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) +$$

$$\ldots + 4f(x_{2n-1} + f(x_{2n})\Big).$$

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