Showing Invisible ObJects

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Abstract

We investigate the properties of mirrored objects in space that have the properties of lenses and the property of invisibility with respect to some fixed direction.

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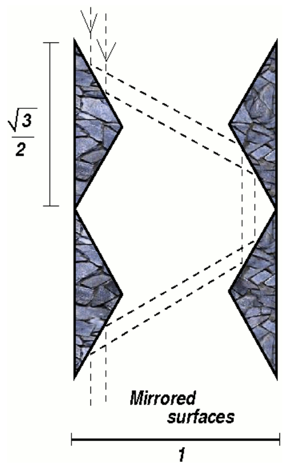
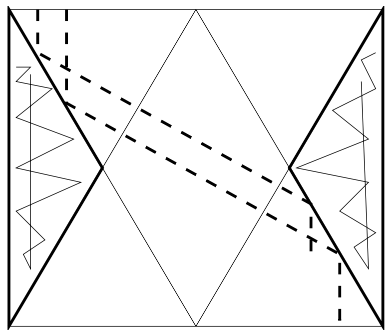
We will say that an object in space (respectively, plane) is *invisible* in a given direction if every ray striking that object from that direction, outside a set of measure zero, can be diverted around the object so that the ray exits the object in the same direction and along the same initial line.



A set of mirrors satisfying the property in the plane.

If we rotate this object about its central axis then we obtain an invisible object in space.

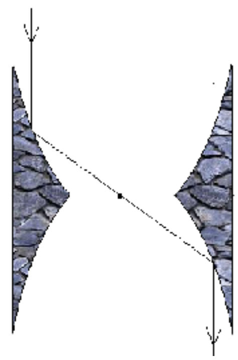
The first element of the object is constructed by inverting an equilateral triangle and placing it with the same central axis in opposition to a copy and filling a mirror along the outside pieces as shown.



A construction of the first element.

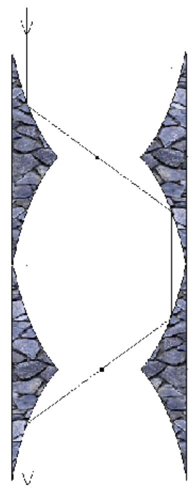
N.B. every ray from infinity in the y direction hits the object at a 30 degree angle and reflects at a thirty degree angle so that it passes to the "parallel point' on the other side. A pair of such elements in series put the ray back into the correct line.

Now we consider a different type of object using parabolas. In this case we construct two congruent parabolas in opposition with major axis along the y-axis such that the focus of one parabola coincides with the focus of the other parabola.



Parabolic mirrors with the same focus.

If we imagine that the focus lies on origin of the xy-plane, then there is a symmetry with respect to the x-axis, y-axis, and the origin. Each ray from infinity parallel to the y-axis which strikes the object must go through the origin and strike the "odd" symmetric point.



Two parabolic mirrors elements in series.

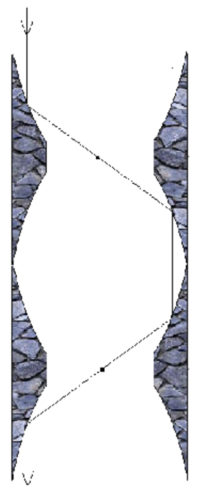
We collect this notion into the following result.

Remark - Parabolas in Series *Consider an object in the plane which is bounded by*

$$-a \leq x \leq -2b, \ y = \frac{1}{b}(\frac{x}{2})^2 - b, \ y = -\frac{1}{b}(\frac{x}{2})^2 + b, \ 2b \leq x \leq a.$$

Consider the same object translated downward in the y *direction by* $2b - a^2/2$ *units. We assume that the object acts as a mirror along the parabolic curves. If a ray from infinity parallel to the* y*-axis strikes the object on one of the parabolic curves at a point* (x_o, y_o)*, then the ray is reflected so that the ray exits parallel to the* y*-axis at a point* $(x_o, 2b - a^2/2 - y_o)$*.*

Corollary *An object constructed by rotating the object in the remark above about the symmetric axis will be invisible with respect to the* y*-direction.*



Two cut away parabolic mirrors in series.

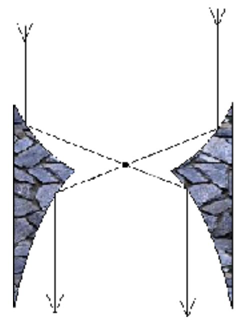
N. B. observe that an invisible object can also be created from the first example i.e., the equilateral triangle example, using this concept.

Remark - Cut away Parabolas in Series *Consider an object in the plane which is bounded by*

$$-a \leq x \leq -c,\ y = \frac{1}{b}(\frac{x}{2})^2 - b,\ y = -\frac{1}{b}(\frac{x}{2})^2 + b,\ c \leq x \leq a,$$

where $a > c \geq 2b$. *Consider the same object translated downward in the* y *direction by* $2b - a^2/2$ *units. The object is invisible with respect to the* y*-direction and the object obtained by rotation about the axis of symmetry is invisible with respect to the* y*-direction.*

Now we move on to a different kind of object. Consider the following construction:



Two offset parabolic mirrors with common focus.

Here we consider a case such that the parabolas are not congruent so that the x-axis symmetry breaks down, but the parabolas in opposition still have the same focus.

Recall that the equation of the parabola with focus at $(0, f)$ and with major axis in the y-direction is

$$4fy = x^2, \text{ or, } y = \frac{1}{f}\Big(\frac{x}{2}\Big)^2.$$

Now observe that if the two parabolas have a common focus at $(0, 0)$ then we have

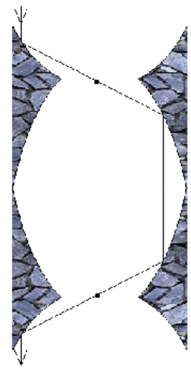
$$\frac{1}{b}\Big(\frac{x}{2}\Big)^2 - b = -\frac{1}{a}(\frac{x}{2})^2 + a,$$

or,

$$(a + b)x^2 = 4(a + b)ab.$$

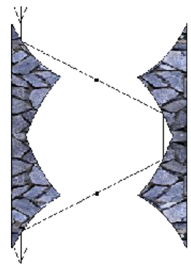
Thus $x = \pm 2\sqrt{ab}$ at the intersection points of the two parabolas.

The idea here is to construct an object of to elements in series so that there is a vertical reflexive symmetry with respect to the two elements.



A pair of lenses in series.

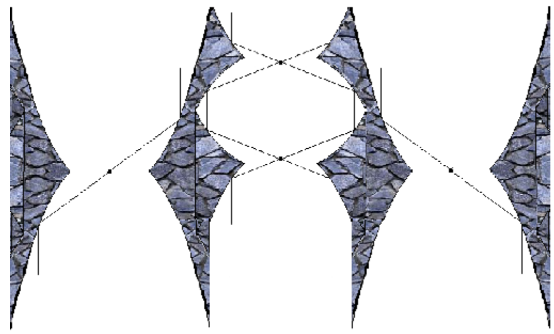
Here we have cut away the unnecessary part reducing the size of the lens.



A length reduced pair of lenses in series.

N. B. The “thinner” parabola must go in the middle section otherwise the middle part must be wider than the original opening to handle the incoming spread of rays

Suppose you wish to construct a system with the incoming rays striking the “thinner” parabola.



A combination lens, where the outside pairs of mirrors must be added.

N.B. The wideness of the middle mirror must exceed the wideness of the incident mirror in order to handle all the incoming rays. Consequently, we must shield this wider section using some other invisible mirror complex. It is possible to reduce contact areas by using a cutaway form.

It is also possible to construct a combination lens using an ellipse in the second stage of the object. The basic design feature is that the ellipse must not be any wider than the incident mouth of the parabola.

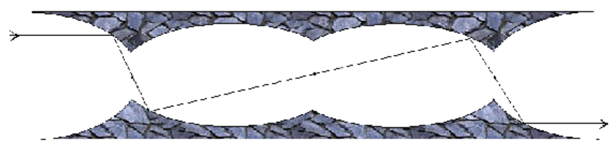


Fig. 3 The combination lens is shown on its side.

The ellipses are congruent (is it possible to do without congruent ellipses in series?) and the all the foci are common. Then the exiting ray is parallel to the y-axis and in line with the initial incoming ray.